

Day-1 [Control Systems].

Unit-1.



Full Video Link

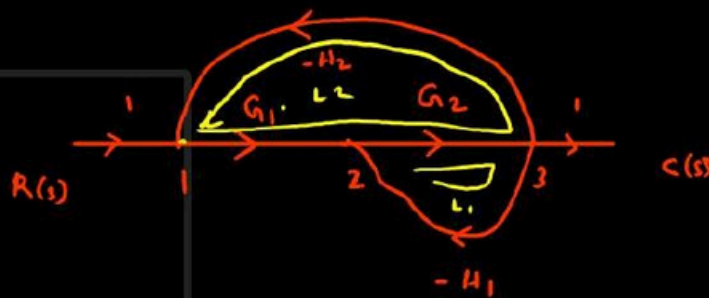


The overall transfer function  $C(s)/R(s)$  of the system shown in the figure below is

(a)  $\frac{G_1 G_2}{1 - G_2 H_2 + G_1 G_2 H_1}$   
 (b)  $\frac{G_1 G_2}{1 + G_2 H_1 - G_1 G_2 H_2}$   
 (c)  $\frac{G_1 G_2}{1 + G_2 H_1 + G_1 G_2 H_2}$   
 (d)  $\frac{G_1 G_2}{G_2 H_1 - G_1 G_2 H_2}$

Maharashtra PSC AE 2019 Paper-I

Ⓒ



T.F. = ?

Forward path  $\Rightarrow P_1 = 1 \times G_1 G_2 \times 1$

$P_1 = G_1 G_2 \quad \Delta_1 = 1$

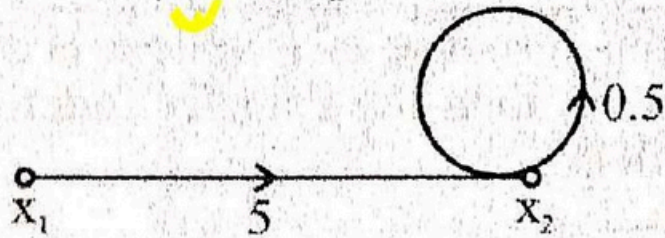
$L_1 = -G_2 H_1 \quad L_2 = -G_1 G_2 H_2$

$$= \frac{G_1 G_2}{1 + G_2 H_1 + G_1 G_2 H_2}$$

$$TF = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2}{1 - \{-G_2 H_1 - G_1 G_2 H_2\}}$$



In the signal flow graph shows,  $X_2 = TX_1$ , where 'T' is equal to



- (a) 2.5
- (c) 5.5

- (b) 5
- (d) 10

Karnataka PSC AE 2016

$$T.F = \frac{5 \times 1}{1 - \{0.5\}} = \frac{5}{1 - 0.5}$$
$$= \frac{5}{0.5} = 10$$

(d)

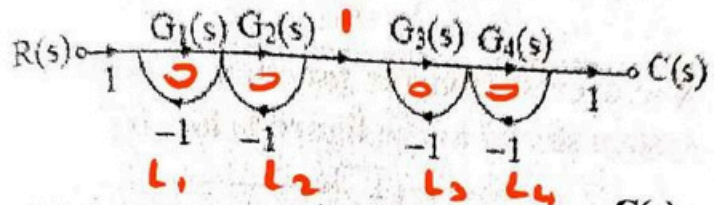
$$X_2 = TX_1 \Rightarrow TF = \frac{X_2}{X_1}$$

$$P_1 = 5 \quad \Delta_1 = 1 \quad L_1 = 0.5$$

Full Video Link



0.5 x



The closed-loop transfer function  $\frac{C(s)}{R(s)}$  of the system represented by the signal flow graph as shown in figure is

- (a)  $\frac{G_1 G_2 G_3 G_4}{(1 + G_1 + G_2)}$  ✗
- (b)  $\frac{G_1 G_2 G_3 G_4}{(1 + G_3 + G_4)}$  ✗
- (c)  $\frac{G_1 G_2 G_3 G_4}{(1 + G_1 + G_2)(1 + G_3 + G_4)}$  ✓
- (d)  $\frac{G_1 G_2 G_3 G_4}{(1 + G_1 + G_2)(1 + G_3 G_4)}$

ESE 2018

$$P_1 = 1 \times G_1 \times G_2 \times 1 \times G_3 \times G_4 = G_1 G_2 G_3 G_4$$

$$T.F = \frac{G_1 G_2 G_3 G_4}{1 - \dots}$$

$$\Delta_1 = 1 - 0 = 1$$

$$L_1 = -G_1 \quad L_2 = -G_2$$

$$L_3 = -G_3 \quad L_4 = -G_4$$

$$L_1 L_3 = -G_1 \times -G_3 = G_1 G_3$$

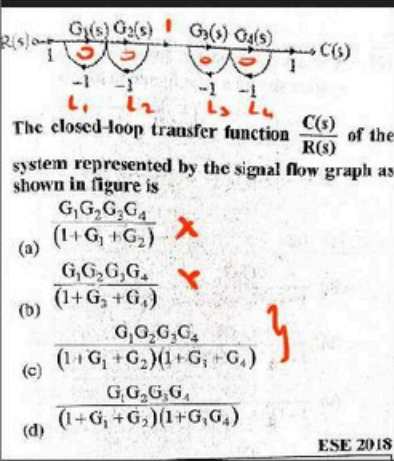
$$L_1 L_4 = -G_1 \times -G_4 = G_1 G_4$$

$$L_2 L_3 = -G_2 \times -G_3 = G_2 G_3$$

$$L_2 L_4 = -G_2 \times -G_4 = G_2 G_4$$

C





$P_1 = 1 \times G_1 \times G_2 \times 1 \times G_3 \times G_4$   
 $= G_1 G_2 G_3 G_4$   
 $\Delta_1 = 1 - 0 = 1$   
 $L_1 = -G_1$     $L_2 = -G_2$   
 $L_3 = -G_3$     $L_4 = -G_4$   
 $L_1 L_3 = -G_1 \times -G_3 = G_1 G_3$   
 $L_1 L_4 = -G_1 \times -G_4 = G_1 G_4$   
 $L_2 L_3 = -G_2 \times -G_3 = G_2 G_3$   
 $L_2 L_4 = -G_2 \times -G_4 = G_2 G_4$

T.F =  $\frac{G_1 G_2 G_3 G_4}{1 - \{ -G_1 - G_2 - G_3 - G_4 \} + \{ G_1 G_3 + G_1 G_4 + G_2 G_3 + G_2 G_4 \}}$

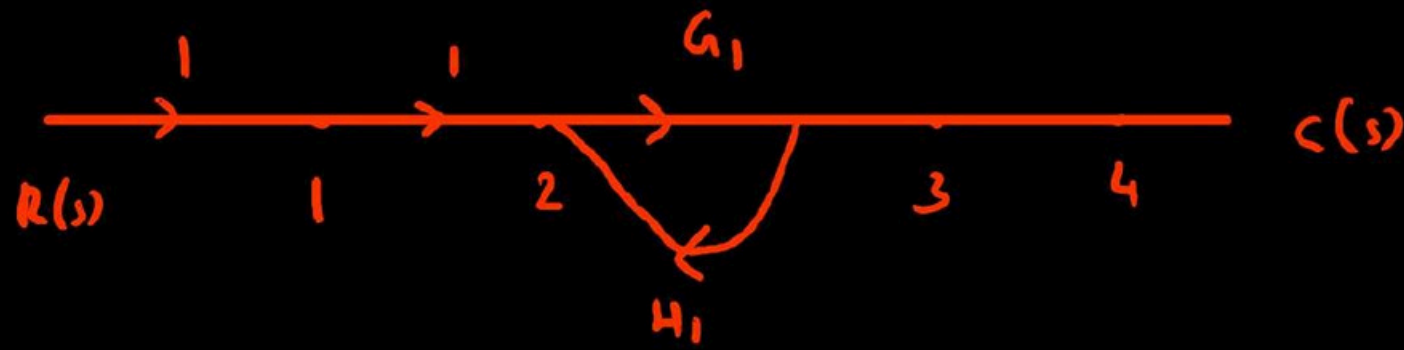
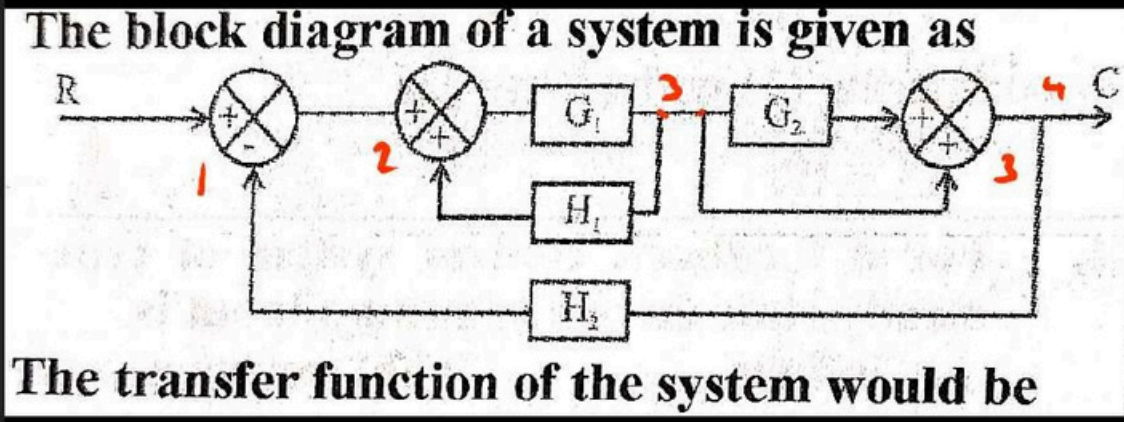
$= \frac{G_1 G_2 G_3 G_4}{1 + G_1 + G_2 + G_3 + G_4 + G_1 G_3 + G_1 G_4 + G_2 G_3 + G_2 G_4}$

$= \frac{G_1 G_2 G_3 G_4}{[1 + G_3 + G_4] + G_1 [1 + G_3 + G_4] + G_2 [1 + G_3 + G_4]}$

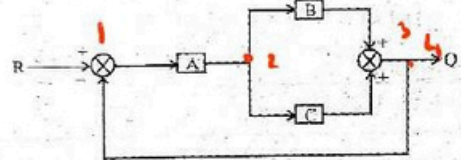
$= \frac{G_1 G_2 G_3 G_4}{[1 + G_3 + G_4] [1 + G_1 + G_2]}$

(c)





The transfer function of the system given below is



- (a)  $\frac{Q}{R} = \frac{A+B+C}{1+AB+AC}$
- (b)  $\frac{Q}{R} = \frac{ABC}{1+ABC}$
- (c)  $\frac{Q}{R} = \frac{AB+AC}{1+AB+AC}$
- (d)  $\frac{Q}{R} = \frac{AB+AC}{ABC}$

UPPCL AE 2014

$P_1 = 1 \times A \times B \times 1 \times 1 \quad \Delta_1 = 1$

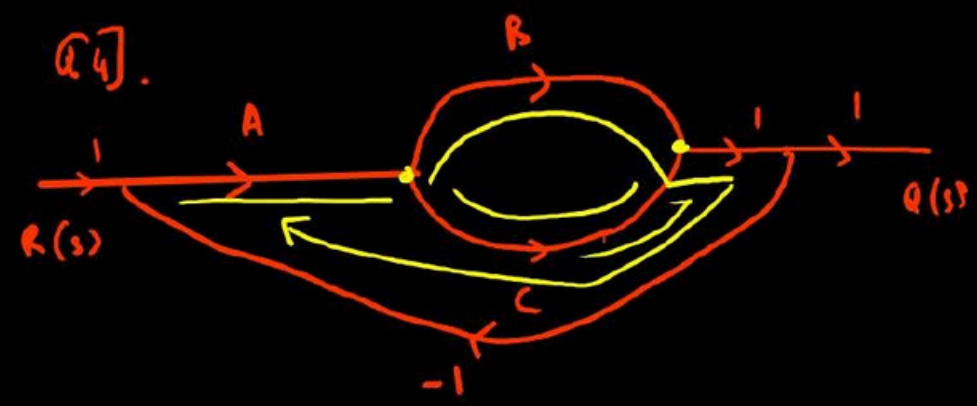
$P_2 = 1 \times A \times C \times 1 \times 1 \quad \Delta_2 = 1$

$L_1 = -AB$

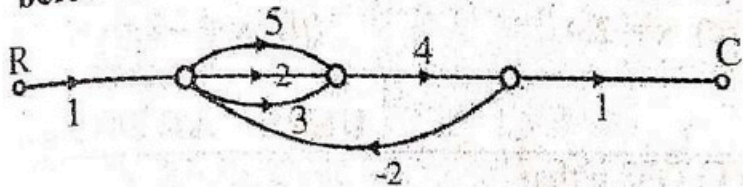
$L_2 = -AC$

TF =  $\frac{AB+AC}{1 - \{-AB-AC\}}$

=  $\frac{AB+AC}{1+AB+AC}$  (C)



The value of  $\frac{C}{R}$  in the signal flow graph shown below is



- (a)  $\frac{28}{27}$   
 (c)  $\frac{40}{81}$

- (b)  $\frac{40}{57}$   
 (d)  $\frac{28}{81}$

UKPSC AE 2007, Paper-I

$$L_1 = 5 \times 4 \times -2 = -40$$

$$L_2 = 2 \times 4 \times -2 = -16$$

$$L_3 = 3 \times 4 \times -2 = -24. \quad \textcircled{C}$$

$$T.F = \frac{20 + 8 + 12}{1 - \{-40 - 16 - 24\}}$$

$$= \frac{40}{1 + 80} = \frac{40}{81}$$

$$Q5]. P_1 = 1 \times 5 \times 4 \quad \Delta_1 = 1$$

$$P_2 = 1 \times 2 \times 4 \times 1 \quad \Delta_2 = 1$$

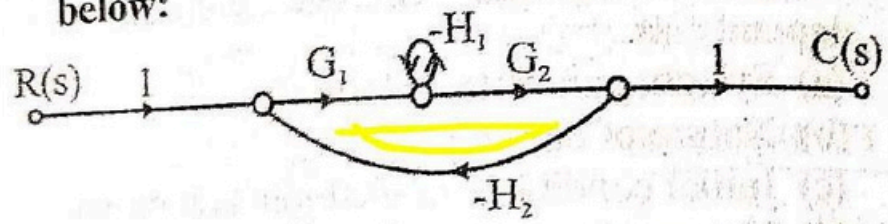
$$P_3 = 1 \times 3 \times 4 \times 1 \quad \Delta_3 = 1$$

Full Video Link



0.5 x

The signal flow graph of a system is given below:



The transfer function of the system is

- |   |  |
|---|--|
| (a) $\frac{G_1(1+H_1)G_2}{1+G_1G_2H_2}$ | (b) $\frac{(G_1+G_2)H_1}{1+G_1G_2H_2}$ |
| (c) $\frac{1+G_1G_2}{1+G_1G_2H_2+H_1}$  | (d) $\frac{G_1G_2}{1+G_1G_2H_2+H_1}$   |

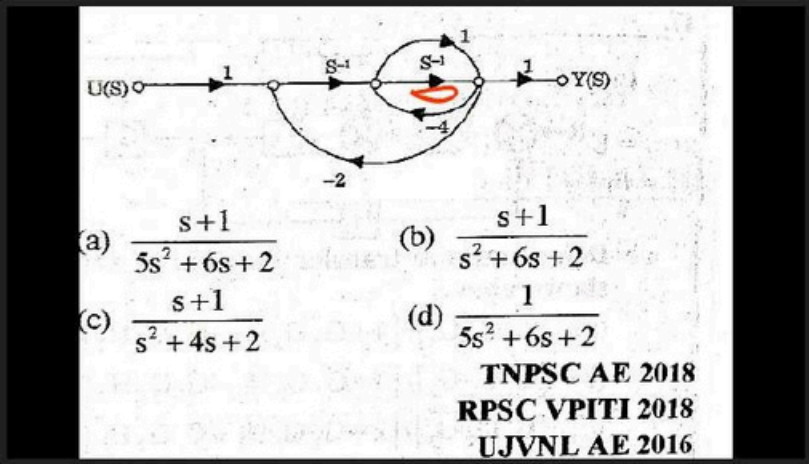
$$TF = \frac{G_1 G_2}{1 - \{-H_1 - G_1 G_2 H_2\}} \textcircled{D}$$

$$= \frac{G_1 G_2}{1 + H_1 + G_1 G_2 H_2}$$

Q.1].  $P_1 = 1 \times G_1 \times G_2 \times 1$      $\Delta_1 = 1 - \{0\}$ .

$L_1 = -H_1$      $L_2 = -G_1 G_2 H_2$





$$L_1 = 1 \times -4 = -4$$

$$L_2 = \frac{1}{s} \times \frac{1}{s} \times -2 = -\frac{2}{s^2}$$

$$L_3 = \frac{1}{s} \times 1 \times -2 = -\frac{2}{s} \quad L_4 = -\frac{4}{s}$$

$$T.F = \frac{1}{s^2} + \frac{1}{s}$$

$$Q.7]. P_1 = 1 \times \frac{1}{s} \times \frac{1}{s} \times 1 \quad \Delta_1 = 1$$

$$P_2 = 1 \times \frac{1}{s} \times 1 \times 1 \quad \Delta_2 = 1$$

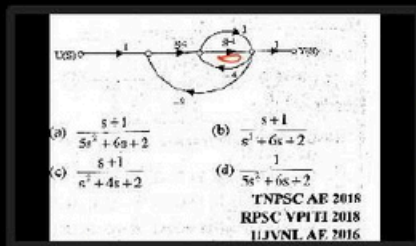
$$1 - \left\{ -4 - \frac{2}{s^2} - \frac{2}{s} - \frac{4}{s} \right\}$$

$$= \frac{1+s}{s^2}$$

$$= \frac{1+s}{s^2} \times \frac{s^2+4s+2}{s^2+4s+2}$$

$$4s^2 + 2 + 2s + 4s$$





$$L_1 = 1 \times -4 = -4$$

$$L_2 = \frac{1}{s} \times \frac{1}{s} \times -2 = -\frac{2}{s^2}$$

$$L_3 = \frac{1}{s} \times 1 \times -2 = -\frac{2}{s}$$

$$L_4 = -\frac{4}{s}$$

$$T.F = \frac{1}{s^2} + \frac{1}{s}$$

Q.7].  $P_1 = 1 \times \frac{1}{s} \times \frac{1}{s} \times 1$   $D_1 = 1$   
 $P_2 = 1 \times \frac{1}{s} \times 1 \times 1$   $D_2 = 1$

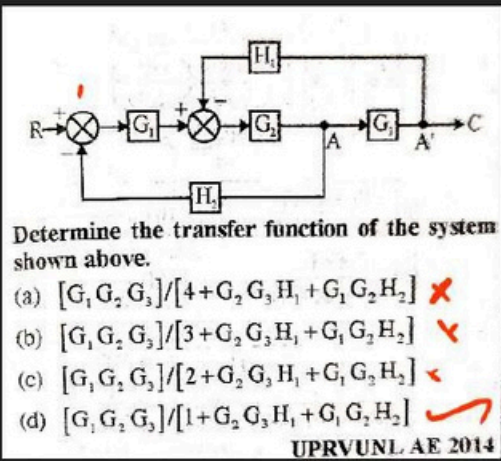
$$1 - \left\{ -4 - \frac{2}{s^2} - \frac{2}{s} - \frac{4}{s} \right\}$$

$$= \frac{s+1}{5s^2 + 2s + 2} //.$$

$$= \frac{1+s}{s^2} = \frac{1+s}{s^2 + 4s^2 + 2s + 2} = \frac{1+s}{5s^2 + 2s + 2} //.$$

$$= \frac{1+s}{s^2} = \frac{1+s}{5s^2 + 2s + 2} //.$$





$P_1 = G_1 G_2 G_3 \quad \Delta_1 = 1$

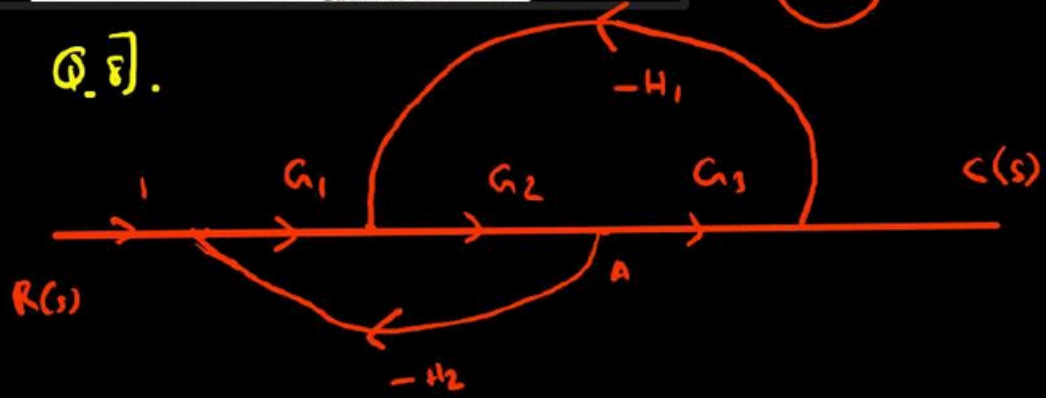
$L_1 = -G_1 G_2 H_2$

$L_2 = -G_2 G_3 H_1$

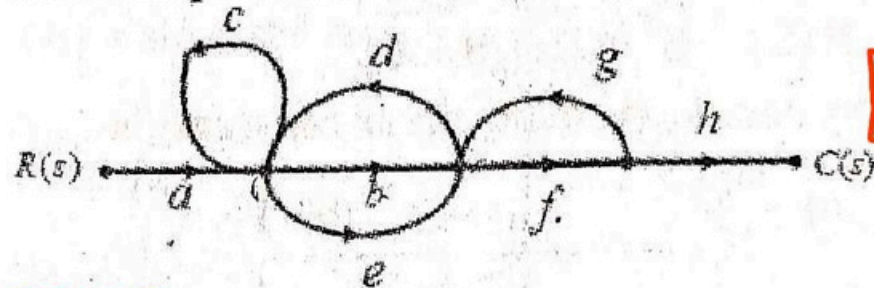
T.F. =  $\frac{G_1 G_2 G_3}{1 - \{ -G_1 G_2 H_2 - G_2 G_3 H_1 \}}$

(D)

=  $\frac{G_1 G_2 G_3}{1 + G_1 G_2 H_2 + G_2 G_3 H_1}$



In a given signal flow graph the number of non-touching loops pairs and number of forward paths are respectively are:



$L_1, L_4$

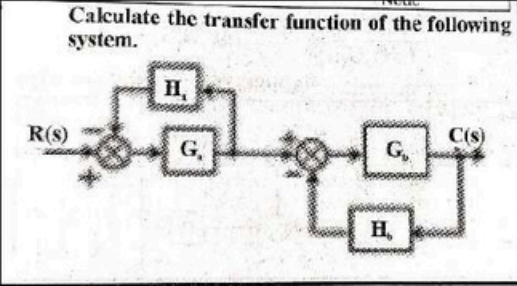
- (a) 1, 2
- (c) 3, 1

- (b) 1, 3
- (d) 2, 5

UPPCL AE 18-05-2016

Q.7.

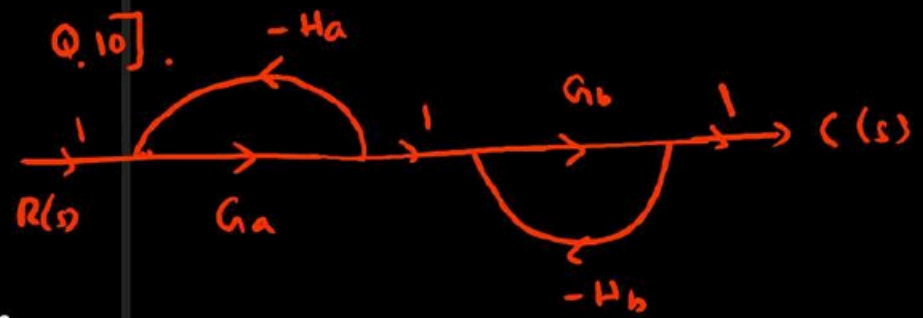




DMRC AM 2019

- A).  $\frac{G_1 G_2}{1 + G_1 H_1 + G_2 H_2 + G_1 G_2 H_1 H_2}$
- B).  $\frac{G_1 G_2}{1 - G_1 H_1 + G_2 H_2 + G_1 G_2 H_1 H_2}$
- C).  $\frac{G_1 G_2}{1 + G_1 G_2 + H_1 H_2}$
- D).  $\frac{G_1 G_2}{1 + G_1 G_2 H_2 + G_2 H_1}$

**A**



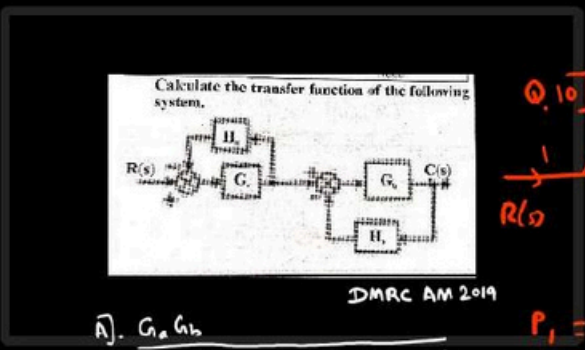
$P_1 = G_1 G_2 \quad D_1 = 1$

$L_1 = -G_1 H_1 \quad L_2 = -G_2 H_2$

$L_1 L_2 = G_1 G_2 H_1 H_2$

T.F =  $\frac{G_1 G_2}{1 - \{ -G_1 H_1 - G_2 H_2 \} + \{ G_1 G_2 H_1 H_2 \}}$





- A).  $\frac{G_a G_b}{1 + G_a H_a + G_b H_b + G_a G_b H_a H_b}$
- B).  $\frac{G_a G_b}{1 - G_a H_a - G_b H_b + G_a G_b H_a H_b}$
- C).  $\frac{G_a G_b}{1 + G_a G_b + H_a H_b}$
- D).  $\frac{G_a G_b}{1 + G_a G_b H_b + G_b H_a}$

**A**



$P_1 = G_a G_b \quad \Delta_1 = 1$   
 $L_1 = -G_a H_a \quad L_2 = -G_b H_b$   
 $L_1 L_2 = G_a G_b H_a H_b$

T.F =  $\frac{G_a G_b}{1 - \{ -G_a H_a - G_b H_b \} + \{ G_a G_b H_a H_b \}}$   
 $= \frac{G_a G_b}{1 + G_a H_a + G_b H_b + G_a G_b H_a H_b}$



Transfer function  $\frac{C(s)}{R(s)}$  of the system shown in the figure here is:

(a)  $\frac{G_a G_b}{H_a (1 + G_a G_b H_b)}$  (b)  $\frac{G_a G_b}{1 + G_a G_b H_a H_b}$   
 (c)  $\frac{G_a G_b H_b}{H_a (1 + G_a G_b H_b)}$  (d)  $\frac{G_a H_b}{H_a (1 + G_a G_b H_b)}$

DMRC AM 2020

G-2019, 2020, 2021, 2022  
 EE, EC, IN

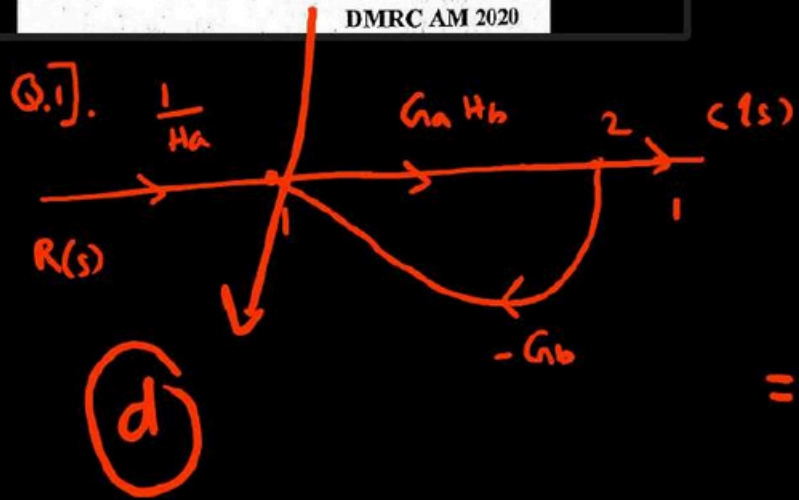
$$P_1 = \frac{1}{H_a} \times G_a H_b \times 1 \quad \Delta_1 = 1$$

$$L = -G_a G_b H_b$$

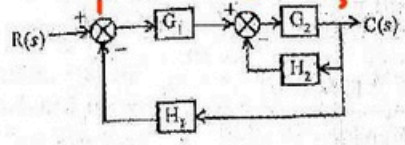
$$T.F = \frac{G_a H_b}{H_a}$$

$$1 - \{-G_a G_b H_b\}$$

$$= \frac{G_a H_b}{H_a \{1 + G_a G_b H_b\}}$$

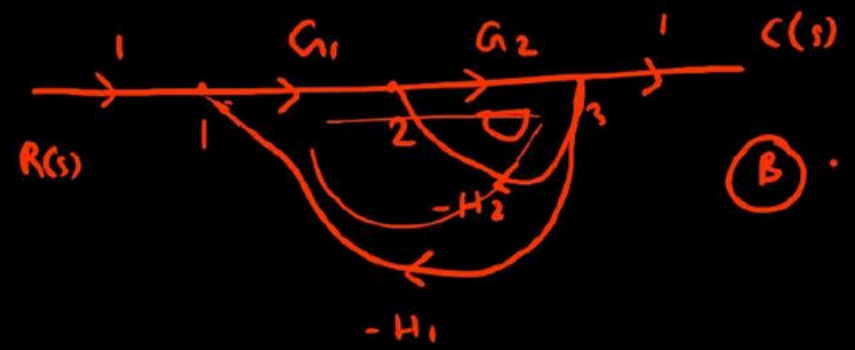


The overall transfer function of the system shown below is



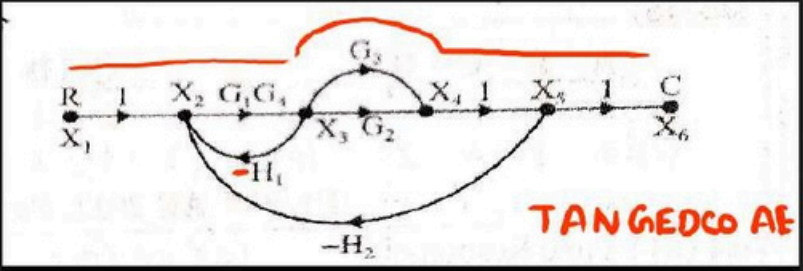
- A).  $\frac{G_1 G_2}{1 + G_1 H_1 + G_1 G_2 H_2}$
- B).  $\frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 H_1}$
- C).  $\frac{G_1 G_2}{1 + G_2 H_2 - G_1 G_2 H_1}$
- D).  $\frac{G_1 G_2}{1 - G_2 H_2 - G_1 G_2 H_1}$

$P_1 = G_1 G_2 \quad \Delta_1 = 1$   
 $L_1 = -G_2 H_2 \quad L_2 = -G_1 G_2 H_1$   
 $T.F = \frac{G_1 G_2}{1 - \{-G_2 H_2 - G_1 G_2 H_1\}}$   
 $= \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 H_1}$



Full Video Link





$$P_1 = G_1 G_4 G_2 \quad \Delta_1 = 1$$

$$P_2 = G_1 G_4 G_3 \quad \Delta_2 = 1.$$

$$T.F = \frac{G_1 G_2 G_4 + G_1 G_3 G_4}{1 + G_1 G_4 G_2 H_2 + G_1 G_4 H_1 + G_1 G_4 G_3 H_2^2}$$

A)  $\frac{G_1 G_2 G_4 + G_1 G_3 G_4}{1 + G_1 G_4 H_1 + G_1 G_2 G_4 H_2 + G_1 G_3 G_4 H_2}$  (A)

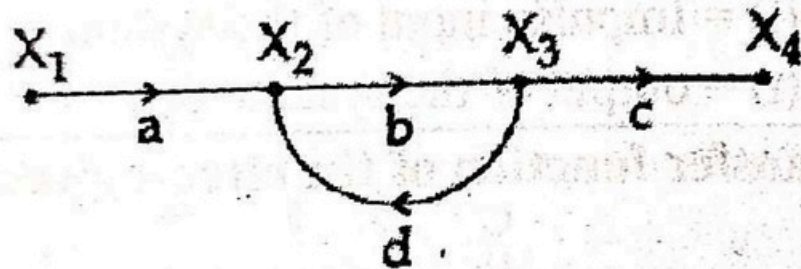
B)  $\frac{G_1 G_2 G_4 + G_1 G_3 G_4}{1 - G_1 G_4 H_1 + G_1 G_2 G_4 H_2 + G_1 G_3 G_4 H_2}$  (B)

C)  $\frac{G_1 G_2 G_4 + G_1 G_3 G_4}{1 - G_1 G_4 H_1 - G_1 G_2 G_4 H_2 + G_1 G_3 G_4 H_2}$

D)  $\frac{G_1 G_2 G_4 + G_1 G_3 G_4}{1 - G_1 G_4 H_1 - G_1 G_2 G_4 H_2 - G_1 G_3 G_4 H_2}$



The total gain  $\frac{X_4}{X_1}$  of the signal flow graph shown in figure is .....



(a)  $\frac{abcd}{1-bd}$

(b)  $\frac{abc}{1+bd}$

(c)  $\frac{abc}{1-bd}$

(d)  $\frac{abc}{d}$

TRB Poly. Lect. 2017

T.F. =  $\frac{abc}{1-bd}$



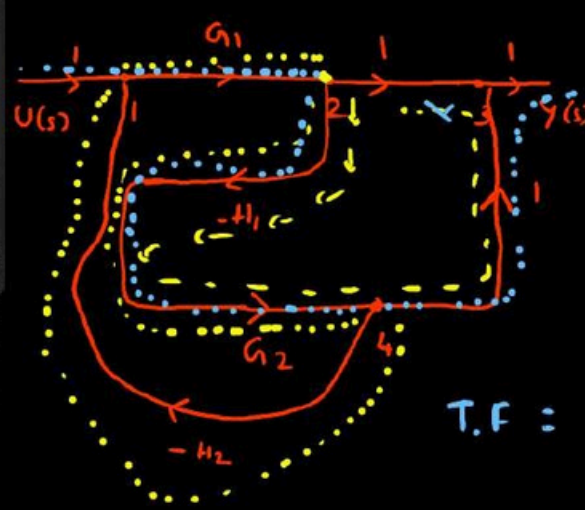
For the block diagram shown, the ratio  $Y(s)/U(s)$

(a)  $\frac{G_1(1+H_1G_2)}{1-G_1H_2G_2H_1}$       (b)  $\frac{G_1(1-H_1G_2)}{1-G_1H_2G_2H_1}$   
 (c)  $\frac{G_1(1-H_1G_2)}{1+G_1H_2G_2H_1}$       (d)  $\frac{G_1(1+H_1G_2)}{1+G_1H_2G_2H_1}$

TNPSC AE 2018

**B**

$$T.F = \frac{G_1 [1 - G_2 H_1]}{1 - G_1 G_2 H_1 H_2}$$



$P_1 = G_1$        $\Delta_1 = 1$

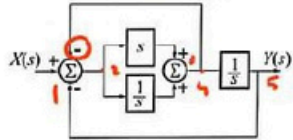
$P_2 = -G_1 G_2 H_1$        $\Delta_2 = 1$

$L = G_1 H_1 G_2 H_2$

T.F =  $\frac{G_1 - G_2 G_1 H_1}{1 - G_1 G_2 H_1 H_2}$



Q.32 The block diagram of a system is illustrated in the figure shown, where  $X(s)$  is the input and  $Y(s)$  is the output. The transfer function  $H(s) = \frac{Y(s)}{X(s)}$  is



- (A)  $H(s) = \frac{s^2+1}{s^3+s^2+s+1}$
- (B)  $H(s) = \frac{s^2+1}{s^2+1}$
- (C)  $H(s) = \frac{s^2+1}{s^3+2s^2+s+1}$
- (D)  $H(s) = \frac{s^2+1}{2s^2+1}$

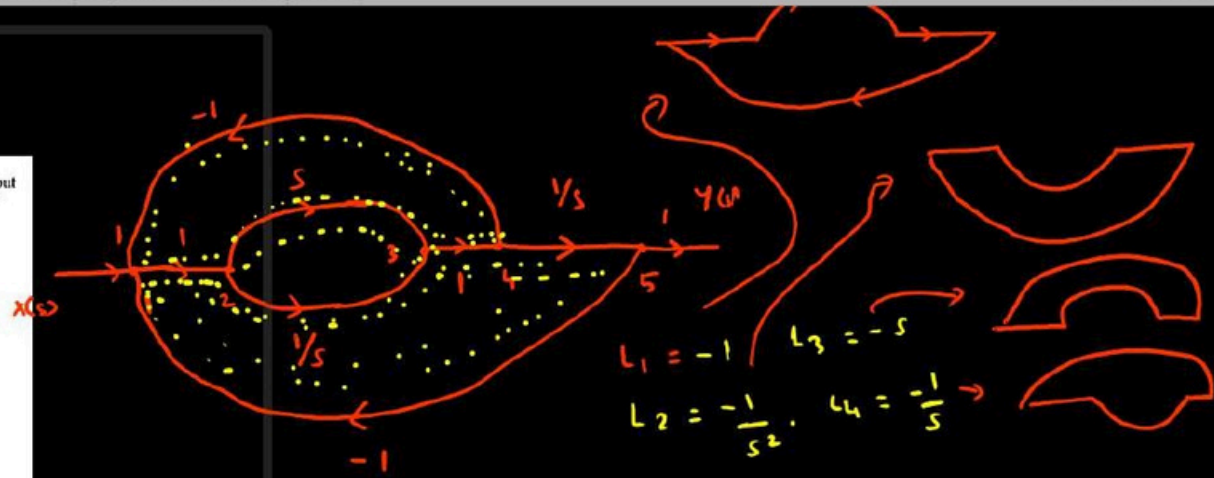
GATE 2019, EC

$p_1 = 1$   
 $p_2 = \frac{1}{s^2}$

$\Delta_1 = 1$   
 $\Delta_2 = 1$

T.F =  $\frac{1 + \frac{1}{s^2}}{1 - \left\{ -1 - s - \frac{1}{s^2} - \frac{1}{s} \right\}}$

T.F =  $\frac{s^2+1}{s^2}$   
 $\frac{2 + s + \frac{1}{s^2} + \frac{1}{s}}$   
 =  $\frac{s^2+1}{s^2}$   
 $\frac{s^2+1}{2s^2 + s^3 + 1 + s}$

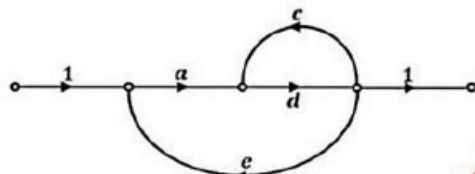


13



Q.No. 13 Which of the options is an equivalent representation of the signal flow graph shown here?

$\frac{ad}{1-cd} \times e$



$P_1 = ad \quad \Delta_1 = 1$   
 $T.F = \frac{ad}{1 - \{cd + ade\}} = \frac{ad}{1 - cd - ade}$

- (A)
- (B)
- (C)
- (D)

$\Rightarrow$  (C)

$\Rightarrow T.F = \frac{ad}{1-cd} \times \frac{1}{1 - \left\{ \frac{ade}{1-cd} \right\}} = \frac{ad}{1-cd-ade}$

GATE 2020, EE

Full Video Link



Q.11 For the closed-loop system shown, the transfer function  $\frac{E(s)}{R(s)}$  is

(A)	$\frac{G}{1+GH}$
(B)	$\frac{GH}{1+GH}$
(C)	$\frac{1}{1+GH}$
(D)	$\frac{1}{1+G}$

UPPCL AE 2019. GATE 2021 EE.



$$E(s) = R(s) - A(s) \quad \text{--- (1)}$$

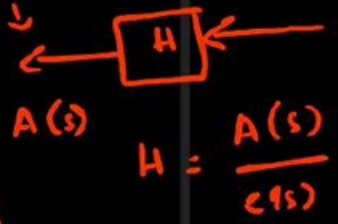
$$E(s) = R(s) - H(s) C(s)$$

$$E(s) = R(s) - H(s) \cdot E(s) \cdot G(s)$$

$$E(s) + H(s) \cdot E(s) \cdot G(s) = R(s)$$

$$E(s) [1 + G(s)H(s)] = R(s)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$



$$H = \frac{A(s)}{C(s)} \Rightarrow A(s) = H C(s) \quad \text{--- (2)}$$

$$G = \frac{C(s)}{E(s)} \Rightarrow C(s) = E(s) \cdot G(s) \quad \text{--- (3)}$$



**Q.13** The block diagram of a feedback control system is shown in the figure.

The transfer function  $\frac{Y(s)}{X(s)}$  of the system is

(A)	$\frac{G_1 + G_2 + G_1 G_2 H}{1 + G_1 H}$
(B)	$\frac{G_1 + G_2}{1 + G_1 H + G_2 H}$
(C)	$\frac{G_1 + G_2}{1 + G_1 H}$
(D)	$\frac{G_1 + G_2 + G_1 G_2 H}{1 + G_1 H + G_2 H}$

GATE 2021, EC

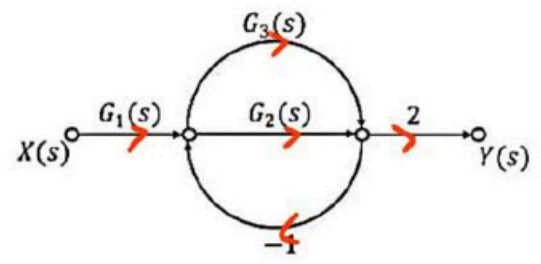


$P_1 = G_1$   
 $P_2 = G_2$   
 $\Delta_1 = 1$   
 $\Delta_2 = 1$   
 $T.F = \frac{G_1 + G_2}{1 - \{-G_1 H\}}$   
 $= \frac{G_1 + G_2}{1 + G_1 H}$

Full Video Link



Q.37 The signal flow graph of a system is shown. The expression for  $Y(s)/X(s)$  is \_\_\_\_\_



- (A)  $\frac{2G_1(s)G_2(s) + 2G_1(s)G_3(s)}{1 + G_2(s) + G_3(s)}$  **(A)**
- (B)  $2 + G_1(s) + G_3(s) + \frac{G_2(s)}{1 + G_2(s)}$
- (C)  $G_1(s) + G_3(s) - \frac{G_2(s)}{2 + G_2(s)}$
- (D)  $\frac{2G_1(s)G_2(s) + 2G_1(s)G_3(s) - G_1(s)}{1 + G_2(s) + G_3(s)}$

Q.10]  $P_1 = 2G_1G_2$       $\Delta_1 = 1$   
 $P_2 = 2G_1G_3$       $\Delta_2 = 1$

T.F =  $\frac{2G_1G_2 + 2G_1G_3}{1 - \{-G_2 - G_3\}}$   
 $= \frac{2G_1G_2 + 2G_1G_3}{1 + G_2 + G_3}$

GATE 2022, IN

Full Video Link



## Unit - 2 Stability.

### Complex Poles.

LHP  $\rightarrow$  BIBO Stable.

RHP  $\rightarrow$  Unstable.

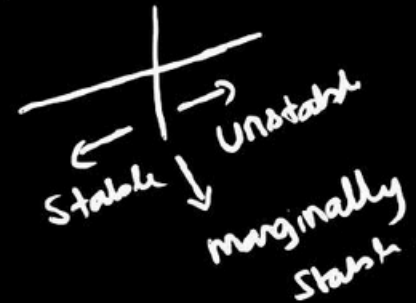
Imaginary poles: Marginally Stable.

Repeated Poles: Unstable System.

- \* System Stable in LHP because we get bounded output value.
- \* System Unstable in RHP because we get unbounded output value.
- \* System Marginally Stable in Zero point because we get unbounded area of output.

\* No. of Sign changes = no. of poles lies in RHP of S-plane.

RH



Full Video Link



The positive value of K for which  $\left[1 + \frac{K}{(s+1)(s+2)}\right]$  will have zeroes on the right-half of the complex s-plane is

- (a) 20 ✗  
 (b) No such K exists  
 (c) 0.1 ✗  
 ✗ (d) 10

UPPCL AE 01-01-2019 Shift I

CE :  $(s+1)(s+2) + k = 0$

open

$1s^2 + 3s + (2+k) = 0$

$2 + 0.1 = 2.1$   
 $2+k$   
 $22$   
 $12$

$$\frac{3(2+k) - 0}{3} = \frac{3(2+k)}{3}$$

$1 + G(s)H(s)$

$G(s) = \frac{k}{(s+1)(s+2)}$

OLTF =  $\frac{N}{D}$  CLTF = ?  
 $= \frac{N}{D+N}$

CLTF =  $\frac{k}{(s+1)(s+2) + k} = CE$



Consider a polynomial,  $s^3 - 2s^2 + s + 1$ . The number of roots of the polynomial on the open left half of complex s-plane is

- (a) Less than or equal to 3. ✗
- (b) Strictly less than 3. ✓
- (c) Equal to 3. ✗
- (d) Strictly greater than 3. ✗

UPPCL AE 01-01-2019 Shift II

Q.2]

$$s^3 - 2s^2 + s + 1$$

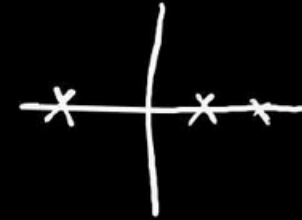
$s^3$	1	1
$s^2$	-2	1
$s^1$	$\frac{3}{2}$	0
$s^0$	1	

Signs: +ve, -ve, +ve, +ve  
 Sign changes: 2

(b)

2 Sign changes.

no. of RH of s-plane = 2.



$$\frac{-2-1}{-2} = \frac{3}{2}$$

$$\frac{3/2 - 0}{3/2}$$



0.4 x

Full Video Link



Forward path transfer function of a unity feedback system is

$$G(S) = \frac{K}{(s+5)(s+10)(s+15)}$$

Select appropriate value of K for the system to be oscillatory.

- (a) 4000
- (b) 5250
- (c) 2400
- (d) 7500

DMRC AM 2020

System  $\rightarrow$  Marginally Stable  $\rightarrow$  Oscillatory.

(d)

Sol: C.E =  $(s+5)(s+10)(s+15) + K = 0$

$$(s^2 + 15s + 50)(s+15) + K = 0$$

$$s^3 + 15s^2 + 50s + 15s^2 + 225s + 750 + K = 0$$

$$s^3 + 30s^2 + 275s + (750 + K) = 0$$

$s^3$	1	275
$s^2$	30	$750 + K$
$s^1$	$\frac{7500 - K}{30}$	
$s^0$	$750 + K$	

$K = 7500$

$$\begin{array}{r} 15 \\ \times 5 \\ \hline 75 \end{array}$$

$$\begin{array}{r} 225 \\ \times 50 \\ \hline 11250 \end{array}$$

$$\begin{array}{r} 275 \times 2 \\ \hline 550 \\ \hline 8250 - 750 - K \\ \hline 7500 - K \end{array}$$

$$\begin{array}{r} 8250 - (750 + K) \\ \hline 7500 - K \end{array}$$



A system with characteristic equation

$$F(s) = s^4 + 6s^3 + 23s^2 + 40s + 50$$

will have closed loop poles such that

- (a) All poles lie in the left half of the s-plane and no pole lies on imaginary axis
- (b) Two poles lie symmetrically on the imaginary axis of the s-plane
- (c) All four poles lie on the imaginary axis of the s-plane
- (d) All four poles lie in the right half of the s-plane

ESE 2019

$$CE = s^4 + 6s^3 + 23s^2 + 40s + 50$$

All poles lies in LHP.

$s^4$	1	23	50
$s^3$	6	40	
$s^2$	$\frac{49}{3}$	50	
$s^1$	$\frac{1060}{49}$		
$s^0$	50		

$$\frac{138 - 40}{6} = \frac{49}{3}$$

$$\frac{49}{4} \cdot 3 = 36.75$$

$$1920 - 300 = 1620$$

$$\frac{1620}{3} = 540$$

$$\frac{49}{3} \cdot 540 = 8820$$

$$\frac{8820 - 900}{49} = \frac{7920}{49}$$



Transfer function of a system is  
 $TF = \frac{s^3 + 2s^2 + 3s + 1}{s^3 + s^2 + 2s + 1}$   
 How <sup>many</sup> roots are lying on the right half side of S-Plane for numerator and denominator for the transfer function?  
 (a) 0, 0 ✓ (b) 1, 0  
 (c) 0, 1 (d) None of the above  
 ISRO Scientist/Engineer 2019

Denominator:  
 $s^3 + s^2 + 2s + 1 = 0$

$s^3$	1	2	
$s^2$	1	1	
$s^1$	1	1	
$s^0$	1	1	

$\frac{2-1}{1}$   
A

numerator  $\Rightarrow s^3 + 2s^2 + 3s + 1 = 0$   
 $\Rightarrow$  NO poles RHP = 0.

$s^3$	1	2	3
$s^2$	2	3	1
$s^1$	5/2	1	
$s^0$	1	1	



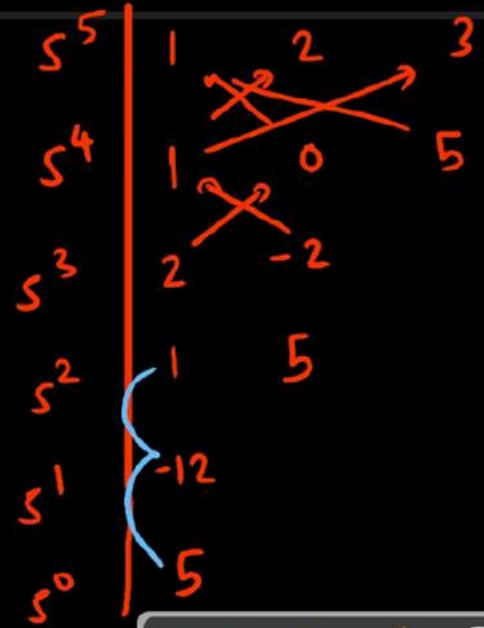
Consider the following characteristic equation:

$$s^5 + s^4 + 2s^3 + 3s + 5 = 0 \Rightarrow s^5 + s^4 + 2s^3 + 0s^2 + 3s + 5 = 0$$

The number of roots in the right half of s-plane will be:

- (a) 4
  - (b) 3
  - (c) 2
  - (d) 1
- UPSC Poly. Lect. 2019

no. of sign changes  
= no. of poles lies in  
 $\frac{2-0}{1} \text{ RHP} = 2$



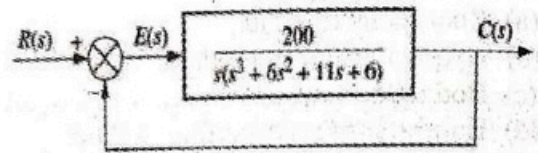
$$\frac{3-5}{1} = -2$$

$$\frac{0-(-2)}{2} = \frac{2}{2} = 1$$

$$\frac{-2-10}{1}$$



Find the number of poles in the left half plane (LHP), the right half plane (RHP) and on the  $j\omega$ -axis for the feedback control system as shown. Is the system stable?



UPPCL AE 04-11-2019 Shift I

**Sol:**

$$G(s) = \frac{200}{s^4 + 6s^3 + 11s^2 + 6s}$$

No. of pole RHP = 2  
No. of poles LHP = 2

$$C.E = s^4 + 6s^3 + 11s^2 + 6s + 200$$

$s^4$	1	11	200
$s^3$	6	6	
$s^2$	10		
$s^1$	-114		
$s^0$	200		

∴ System Unstable

$$\frac{66 - 6}{1} = \frac{60}{1}$$

$$\frac{60 - 1200}{10} = -ve.$$

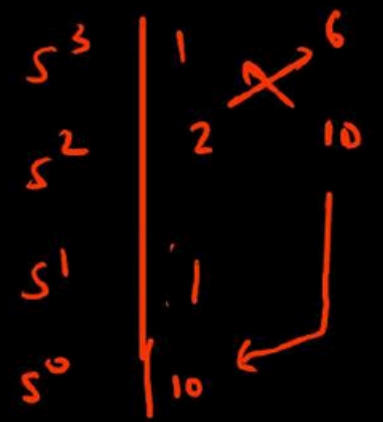
- (a) 1 LHP poles; 3 RHP poles; 0  $j\omega$  poles; system is stable
- (b) 2 LHP poles; 2 RHP poles; 0  $j\omega$  poles; system is unstable
- (c) 2 LHP poles; 2 RHP poles; 0  $j\omega$  poles; system is stable
- (d) 1 LHP poles; 3 RHP poles; 0  $j\omega$  poles; system is stable



Consider a closed-loop system with transfer function  $\frac{C(s)}{R(s)} = \frac{s^2 + 2s + 8}{s^3 + 2s^2 + 6s + 10}$ .  
 Find the number of poles in right half of s plane and in left half of s plane.  
 (a) 3, 1 (b) 0, 3  
 (c) 3, 2 (d) 5, 4  
 DMRC AM 2020

There is no sign changes.  
 No. of poles in RHP = 0. 0, 3  
 No. of poles in LHP = 3.  
 System stable.

C.E =  $s^3 + 2s^2 + 6s + 10 = 0$ .



What is the range of 'K' for which the unity feedback closed loop system with open loop

gain  $G(s) = \frac{K}{s^2(s+a)}$  will be unstable?

(a)  $-a < K < a$

(b)  $K > 0$

(c)  $K = 0$

(d)  $-\infty < K < \infty$

ISRO Scientist/Engineer 2020

$-\infty < k < \infty$

(d)

C.E =  $s^2(s+a) + k$

=  $s^3 + as^2 + 0s + k = 0$

$s^3$	1	0
$s^2$	a	k - 1
$s^1$	$\frac{-k}{a} = -\frac{1}{a}$	
$s^0$	0 + k	1

$\frac{0 - k}{a}$   
 $k = -1$

$k = 1$

System unstable for all k-values.

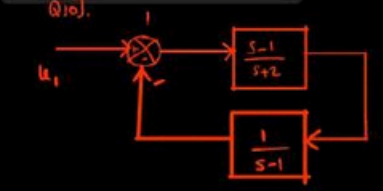
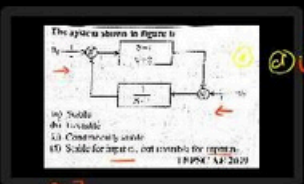
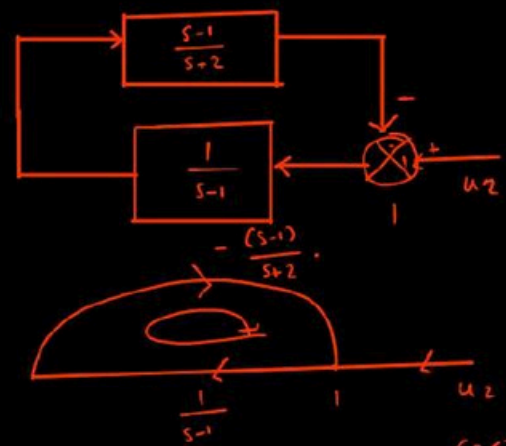
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$$TF = \frac{1}{s-1} \cdot \frac{1}{1 + \frac{1}{s+2}}$$

$$= \frac{1}{s-1} \times \frac{s+3}{s+2}$$

$\therefore$  unstable.



Root locus plot on the s-plane. Poles at  $s=1$  and  $s=-2$ , zero at  $s=-3$ . A closed-loop pole is at  $s=1$ .

$$P_1 = \frac{s-1}{s+2}, \Delta_1 = 1$$

$$L_1 = -\frac{(s-1)}{s+2} \times \frac{1}{s-1}$$

$$TF = \frac{s-1}{s+2}$$

$$P_1 = \frac{1}{s-1}, \Delta_1 = 1$$

$$L_1 = -\frac{(s-1)}{s+2} \times \frac{1}{s-1}$$

$$TF = \frac{s-1}{s+2} \cdot \frac{1}{1 + \frac{(s-1)}{(s+2)(s-1)}}$$

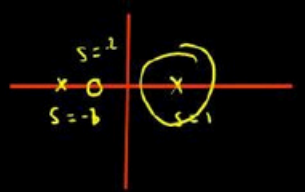
$$= \frac{s-1}{s+2} \cdot \frac{(s+2)(s-1)}{(s+2)(s-1) + (s-1)}$$

$$= \frac{(s-1)^2}{(s+2)(s-1) + (s-1)} = \frac{(s-1)^2}{(s-1)[(s+2)+1]}$$

$$= \frac{s-1}{s+3}$$

$\therefore$  stable.

$$TF = \frac{1}{s-1} \times \frac{s+3}{s+2}$$



$s=1$  } poles.  
 $s=-3$  } zero.  
 $s=-2$  } zero.



Q.No. 23 The loop transfer function of a negative feedback system is

$$G(s)H(s) = \frac{K(s+11)}{s(s+2)(s+8)}$$

The value of  $K$ , for which the system is marginally stable, is \_\_\_\_\_.

GATE 2017, 2020, 2021, 2022.

Q.1]. C.E =  $s(s+2)(s+8) + k(s+11)$ .

$$= s(s^2 + 10s + 16) + ks + 11k.$$

$$= s^3 + 10s^2 + 16s + ks + 11k.$$

$$= s^3 + 10s^2 + s[16+k] + 11k.$$

$$\frac{160-k}{10} = 0 \Rightarrow 160-k=0.$$

$$k=160$$

$$+k = -160$$
$$k = 160.$$

$s^3$	1	$16+k$
$s^2$	10	$11k$
$s^1$	$\frac{160-k}{10}$	0
$s^0$	$11k$	

$$\frac{10(16+k) - 11k}{10}$$

$$\frac{160 + 10k - 11k}{10}$$

Full Video Link



0.5 x

Q.15 The characteristic equation of a linear time-invariant (LTI) system is given by  $\Delta(s) = s^4 + 3s^3 + 3s^2 + s + k = 0$ .  
 The system is BIBO stable if  
 (A)  $0 < k < \frac{12}{9}$  (B)  $k > 3$   
 (C)  $0 < k < \frac{8}{9}$  (D)  $k > 6$

G-2019, EE

Q.2]. C.E =  $\Delta(s) = s^4 + 3s^3 + 3s^2 + s + k = 0$ .

$s^4$		1	3	k
$s^3$		3	1	0
$s^2$		$\frac{8}{3}$	k	
$s^1$		$\frac{8-9k}{8}$		
$s^0$		k		

$\frac{8-9k}{8} > 0$   
 $k > 0$   
 $8-9k > 0$   
 $8 > 9k$

$$\frac{9-1}{3} = \frac{8}{3}$$

$$\frac{\frac{8}{3} - 3k}{\frac{8}{3}} = \frac{8-9k}{8}$$

$0 < k < \frac{8}{9}$

Handwritten notes in red:  
 (C) is circled.  
 25 is circled.  
 10 is circled with an arrow pointing to a bracketed range "100 to 1000".



Q.No. 36 Consider a negative unity feedback system with the forward path transfer function  $\frac{s^2+s+1}{s^3+2s^2+2s+K}$  where  $K$  is a positive real number. The value of  $K$  for which the system will have some of its poles on the imaginary axis is \_\_\_\_\_.

(A) 9  
(B) 8  
(C) 7  
(D) 6

GATE 2020, 2M

Q.3]. I: C.E =  $s^3 + 2s^2 + 2s + k$

$s^3$	1	2	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;"><math>k=4</math></div>
$s^2$	2	$k$	
$s^1$	$\frac{4-k}{2}$		
$s^0$	$k$		

**OLTF**

C.E =  $s^3 + 2s^2 + 2s + k + s^2 + s + 1$   
 $= s^3 + 3s^2 + 3s + 1 + k$

$s^3$	1	3	$\frac{9 - (1+k)}{3}$
$s^2$	3	$1+k$	$\frac{9 - 1 - k}{3}$
$s^1$	$\frac{8-k}{3}$	$\frac{8-k}{3} = 0$	$8-k=0$
$s^0$	$1+k$		<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;"><math>\therefore k=8</math></div>



Q.44 Consider an even polynomial  $p(s)$  given by  
 $p(s) = s^4 + 5s^2 + 4 + K$ ,  
 where  $K$  is an unknown real parameter. The complete range of  $K$  for which  $p(s)$  has all its roots on the imaginary axis is \_\_\_\_\_.

(A)  $-4 \leq K \leq \frac{9}{4}$   
 (B)  $-3 \leq K \leq \frac{9}{2}$   
 (C)  $-6 \leq K \leq \frac{5}{4}$   
 (D)  $-5 \leq K \leq 0$

*Marginally stable.*

**(A)**

GATE 2022 EC, 2M.

Q.4]. C.E =  $s^4 + 0s^3 + 5s^2 + 0s + 4 + k$ .

	$s^4$	$s^2$	
$s^4$	1	5	$4+k$
$s^3$	4	10	
$s^2$	2.5	$4+k$	
$s^1$	$\frac{9-4k}{2.5}$		
$s^0$	$4+k$		

$\frac{9-4k}{2.5} = 0 \Rightarrow 9 = 4k \Rightarrow \frac{9}{4} = k$

$4+k = 0 \Rightarrow k = -4$

$A(s) = s^4 + 5s^2 + 4 + k$ .

$\frac{dA(s)}{ds} = 4s^3 + 10s$ .

$\frac{2.5}{1.8} = \frac{9}{4}$

$-4 \leq k \leq \frac{9}{4}$

$\frac{2.5 - 16 - 4k}{2.5} = \frac{9-4k}{2.5}$



Q.19 The open loop transfer function of a unity gain negative feedback system is given by

$$G(s) = \frac{k}{s^2 + 4s - 5}$$

The range of  $k$  for which the system is stable, is

- (A)  $k > 3$
- (B)  $k < 3$
- (C)  $k > 5$
- (D)  $k < 5$

G-2022 EE 1m.

(C)

$$C.E = s^2 + 4s - 5 + k$$

$$-5 + k > 0 \Rightarrow k > 5$$

$$k > 5$$

$$k > 5$$

$s^2$	1	$-5 + k$
$s^1$	4	$-5 + 4 = -1$
$s^0$	$-5 + k > 0$	$-5 + 1 = -4$

$$-5 + 1 = -4$$

$$7 = 2$$

$$8 = 3$$



TRB 2005.  
 The characteristics equation of a feedback control system is  $s^4 + 20s^3 + 15s^2 + 2s + k = 0$ . Range of k for system to be stable is \_\_\_\_\_

Q.6]. C.E =  $s^4 + 20s^3 + 15s^2 + 2s + k$ .

$s^4$	1	15	k
$s^3$	20	2	
$s^2$	$\frac{149}{10}$	k	
$s^1$	$\frac{298 - 200k}{149}$		
$s^0$	k		

$k > 1.49$

$\frac{298 - 200k}{149} > 0$

$298 - 200k > 0$   
 $+ 200k > + 298$   
 $k > \frac{298}{200}$

$k > \frac{149}{100} = 1.49$

$\frac{149}{20} = 7.45$

$\frac{149 - 100k}{5} < \frac{149}{10}$

$\frac{149 \times 2 - 20k}{10} = \frac{149}{10}$

$\frac{298 - 200k}{149} = \frac{149 - 100k}{5}$



TRB 2012

The characteristic equation of a closed loop

control system is given as  $s^2 + 4s + 16 = 0$ .

The resonant frequency in radians/sec of the system is \_\_\_\_\_

Q.7]. C.E =  $s^2 + 4s + 16 = 0$ .

$s^2$		1	16
$s^1$		4k	
$s^0$		16	

frequency  $\Rightarrow$  oscillate

$\Rightarrow$  marginally stable.

$$A(s) = s^2 + 16 = 0$$

$$s^2 = -16 \quad s = \pm i\sqrt{16}$$

$$s = \pm j\omega = \pm i4$$

$$\omega = 4 \text{ rad/sec}$$

Full Video Link



TRB 2017, EC.

unstable  $k = ?$

For a given system Routh's array is given below  
The system is

$s^4$	1	3	7
$s^3$	1	2	
$s^2$	1	7	
$s^1$	-5		
$s^0$	7		

A]. conditional stable.

B]. stable.

C]. unstable.

D]. marginally stable.

no. of Roots lies in  $4 - 2 = 2$

RHP = 2

1, 2, 3...  $\infty$

$k > 0$

no. of Roots lies in  
LHP = 2.

Full Video Link



TNEB 2018

The characteristics equation of a feedback control system is given by  
 $2s^4 + s^3 + 2s^2 + 5s + 10 = 0$ . The no. of roots in the right half of s-plane are \_\_\_\_\_

$$C.E = 2s^4 + s^3 + 2s^2 + 5s + 10 = 0.$$

$$\boxed{RHP = 2}$$

$$\boxed{LHP = 2}$$

imaginary axis = 0.

$s^4$	2 +ve	2	10
$s^3$	1 +ve	5	
$s^2$	-8 -ve	10	
$s^1$	$\frac{50}{8}$ +ve		
$s^0$	10		

  
$$\frac{2-10}{1}$$
  
$$\frac{-40-10}{-8}$$

Full Video Link



(B)

Q.10. TRB Poly EC 2021, EE 2021

The characteristics equation of a system is given by  $s^3 + 3s^2 + (4+k)s + 2+2k = 0$ . The bound on the value of 'k' for which the system would be stable is:

- A].  $k \geq 10$
- B].  $k > -10$
- C].  $k \geq -1$
- D].  $k \geq 1$

$k > -1$       ~~$k \geq -1$~~   
 $k > -10$

-8 -9 -10

C.F =  $s^3 + 3s^2 + (4+k)s + 2+2k$ .

(B), (C)

$k > -1$

$s^3$	1	$4+k$
$s^2$	3	$2+2k$

$\frac{10+k}{3} > 0$   
 $2+2k > 0$

$2-2=0$   
 $2+0=2$   
 $2+2=4$

$k > -10$   
 $-1 < k < -10$

$\frac{12+3k - 2-2k}{3} = \frac{10+k}{3}$

$\frac{10+k}{3} > 0 \Rightarrow 10+k > 0$

$k > -10$

$2+2k > 0 \Rightarrow 2k > -2$   
 $k > -1$



## State Space Analysis.

- \* No. of State variables = no. of memory circuit elements  
= order of equation.

### Standard Formate:

$$\begin{aligned} \text{State Equation} &\Rightarrow \dot{X} = AX + BU & A &\rightarrow \text{State Matrix} & C &\rightarrow \text{output Matrix} \\ \text{output Equation} &\Rightarrow Y = CX + DU & B &\rightarrow \text{Input Matrix} & D &\rightarrow \text{State Transition Matrix.} \end{aligned}$$

### State Model:

- \* Differential Equation
- \* Transfer Function
- \* SFG
- \* Electric network

$$* TF = C [SI - A]^{-1} B + D$$

$$* CE = [SI - A]^{-1} C$$

$$* \text{State Transition Matrix} = \Phi(t) = e^{At} = L^{-1} \{ [SI - A]^{-1} \}$$

$$* \text{Controllable} \Rightarrow Q_c = [B \ AB] \neq 0$$

$$* \text{Observable} \Rightarrow Q_o = [C^T \ A^T C^T] \neq 0$$

Properties:

$$* \Phi(0) = I$$

$$* \Phi^k(t) = \Phi(kt)$$

$$* \Phi^{-1}(t) = \Phi(-t)$$

$$* \Phi(t_1 + t_2) = \Phi(t_1) \cdot \Phi(t_2)$$

Laplace Transform - Important Formulae:

$$* L[1] = 1/s \quad * L[t^n] = \frac{n!}{s^{n+1}}$$

$$* L[e^{at}] = \frac{1}{s-a}$$

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### Standard Formate:

State Equation  $\Rightarrow \dot{X} = AX + BU$      $A \rightarrow$  State Matrix     $C \rightarrow$  output Matrix  
output Equation  $\Rightarrow Y = CX + DU$      $B \rightarrow$  Input Matrix     $D \rightarrow$  State Transition Matrix.

### State Model:

- \* Differential Equation
- \* Transfer Function
- \* SFG
- \* Electric network

\*  $TF = C [SI - A]^{-1} B + D$     \*  $CE = [SI - A]$

\* State Transition Matrix =  $\Phi(t) = e^{At} = L^{-1} \{ [SI - A]^{-1} \}$

\* Controllable  $\Rightarrow Q_c = [B \ AB] \neq 0$ .

\* Observable  $\Rightarrow Q_o = [C^T \ A^T C^T] \neq 0$ .

### Properties:

\*  $\Phi(0) = I$ .

\*  $\Phi^k(zt) = \Phi(kt)$ .

\*  $\Phi^{-1}(t) = \Phi(-t)$ .

\*  $\Phi(t_1 + t_2) = \Phi(t_1) \cdot \Phi(t_2)$ .

### Laplace Transform - Important Formulae:

\*  $L[1] = 1/s$     \*  $L[t^n] = \frac{n!}{s^{n+1}}$

\*  $L[t^n] = \frac{\Gamma_{n+1}}{s^{n+1}}$     \*  $L[e^{at}] = \frac{1}{s-a}$

\*  $L[e^{-at}] = \frac{1}{s+a}$     \*  $L[\sin at] = \frac{a}{s^2 + a^2}$

\*  $L[\sinh at] = \frac{a}{s^2 - a^2}$     \*  $L[\cosh at] = \frac{s}{s^2 - a^2}$

\*  $L[\cosh at] = \frac{s}{s^2 - a^2}$     \*  $\Gamma_{n+1} = n\Gamma_n$     \*  $\Gamma_{1/2} = \pi$





The transfer function of a system described by the state equations  $\dot{x}(t) = -2x(t) + 2u(t)$  and  $y(t) = 0.5x(t)$  is

- (a)  $0.5/(s+2)$       (b)  $0.5/(s-2)$   
(c)  $1/(s+2)$       (d)  $1/(s-2)$

Rajasthan Nagar Nigam AE 2016, Shift-II

Q2].  $\dot{x}(t) = -2x(t) + 2u(t)$ .

$$y(t) = 0.5x(t).$$

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t).$$

(c)

$$SI = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}_{2 \times 2} [SI - A]^{-1} = \frac{\text{adj} [SI - A]}{|SI - A|}$$

$$A = -2 \quad C = 0.5 \quad [SI - A] = [s + 2]$$

$$B = 2$$

$$\text{T.F.} = C [SI - A]^{-1} B$$

$$= 0.5 [s + 2]^{-1} 2$$

$$= \frac{1}{s + 2}$$

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0.4 x

The transfer function of the following state model of an LTI system with zero initial condition is-

$$\frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 6y(t) = x(t)$$

(a)  $\frac{1}{(s+1)(s+2)(s+3)}$  (b)  $\frac{1}{(s+1)(s+2)(s+2)}$   
 (c)  $\frac{1}{(s+1)(s+2)(s+4)}$  (d)  $\frac{1}{(s+3)(s-1)(s-2)}$

MPPSC AE 2014

(A)

$3 \times 3 \Rightarrow A^{-1} \Rightarrow \text{sm.}$   
 $A, B, C$   
 $TF = C [sI - A]^{-1} B$   
 $Y(0) = Y'(0) = 0.$

Q.3].  $s^3 Y(s) + 6s^2 Y(s) + 11s Y(s) + 6Y(s) = X(s).$

$Y(s) [s^3 + 6s^2 + 11s + 6] = X(s).$

$\frac{Y(s)}{X(s)} = \frac{1}{s^3 + 6s^2 + 11s + 6}$

$$\begin{array}{ccc|c} 1 & 6 & 11 & 6 \\ 0 & -1 & -5 & -6 \\ \hline 1 & 5 & 6 & 6 \end{array}$$

$x^2$

$s = -1 \quad -1 + 6 - 11 + 6 =$

$$\begin{array}{c|c} 6 & 5 \\ \hline 3 & 2 \end{array}$$

$x^2 + 5x + 6 = 0.$   
 $x = -3, -2, -1$

$= \frac{1}{(s+3)(s+2)(s+1)}$



Which of the following statements is correct for the system?

$$\dot{X} = \begin{bmatrix} 2 & 3 \\ 0 & -5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U \quad \dot{X} = AX + BU$$

- (a) The system is controllable but unstable
- (b) The system is uncontrollable but unstable
- (c) the system is controllable but stable
- (d) The system is uncontrollable but stable

UPSC JWM 2017

Q.4]. Controllable =  $[B \ AB] \neq 0$ .

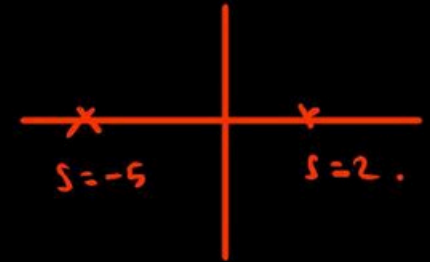
(B)

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Q_c = \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = 0 \quad \text{uncontrollable.}$$

$$C.E = |sI - A|$$

$$sI = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 0 & -5 \end{bmatrix}$$



$$|sI - A| = \begin{vmatrix} s-2 & -3 \\ 0 & s+5 \end{vmatrix} = (s-2)(s+5)$$

$s=2$     $s=-5$

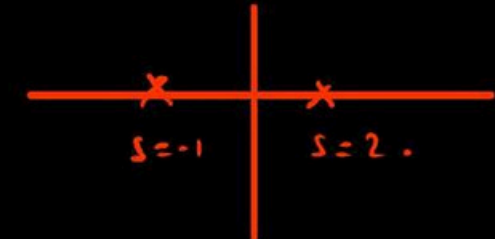
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The system  $\dot{X}(t) = Ax(t) + Bu(t)$  with  
 $A = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$  is:  
 (a) unstable and uncontrollable  
 (b) unstable but controllable  
 (c) stable but uncontrollable  
 (d) stable and controllable  
 BHEL ET 2019

C.E =  $s^2 - s - 2$   
 $AB = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$   
 $\begin{matrix} s^2 & | & 1 & -2 \\ s^1 & | & -1 & \\ s^0 & | & -2 & \end{matrix}$

System unstable.



$Q_c = \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix} = 0 - 2 \neq 0 \Rightarrow$  Controllable.

C.E =  $|sI - A| = \begin{vmatrix} s+1 & -2 \\ 0 & s-2 \end{vmatrix} = (s+1)(s-2)$   
 $s = -1 \quad s = 2$

(B)



The transfer function for the state variable representation  $\dot{X} = AX + BU, Y = CX + DU$  is given by:

- (a)  $D + C(sI - A)^{-1}B$
- (b)  $D(sI - A)^{-1}C + B$
- (c)  $D(sI - A)^{-1}B + C$
- (d)  $B(sI - A)^{-1}C + D$

BHEL ET 2019

(A)

$$T.F = C [sI - A]^{-1} B + D.$$

$$= B [sI - A]^{-1} C + D \quad \times$$



Find the state transition matrix  $\Phi(t)$  if  $A =$

$$\begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$$

- (a)  $\begin{bmatrix} (2e^{-t} - e^{-2t}) & (e^{-t} - e^{-2t}) \\ (-2e^{-t} + 2e^{-2t}) & (-e^{-t} + 2e^{-2t}) \end{bmatrix}$
- (b)  $\begin{bmatrix} (2e^{-t} - e^{-2t}) & (-2e^{-t} + 2e^{-2t}) \\ (e^{-t} + e^{-2t}) & (-e^{-t} + 2e^{-2t}) \end{bmatrix}$
- (c)  $\begin{bmatrix} (2e^{-t} - e^{-2t}) & (-2e^{-t} + 2e^{-2t}) \\ (e^{-t} - e^{-2t}) & (-e^{-t} + 2e^{-2t}) \end{bmatrix}$
- (d)  $\begin{bmatrix} (2e^{-t} + e^{-2t}) & (-2e^{-t} + 2e^{-2t}) \\ (e^{-t} - e^{-2t}) & (-e^{-t} + 2e^{-2t}) \end{bmatrix}$

OMC Deputy manager 2019

2M

$$[sI - A]^{-1} = \begin{bmatrix} \frac{s+3}{(s+2)(s+1)} & \frac{-2}{(s+2)(s+1)} \\ \frac{1}{(s+2)(s+1)} & \frac{s}{(s+2)(s+1)} \end{bmatrix}$$

Q7].  $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

$$\Phi(t) = L^{-1} \{ [sI - A]^{-1} \}$$

$$[sI - A] = \begin{bmatrix} s & 2 \\ -1 & s+3 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix}}{s(s+3)+2}$$

$$\Phi(t) = L^{-1} \left[ \begin{array}{cc} 2e^{-t} - e^{-2t} & 2e^{-2t} - 2e^{-t} \\ e^{-t} - e^{-2t} & 2e^{-t} - e^{-2t} \end{array} \right]$$

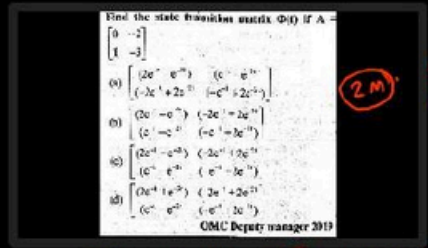
$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix}}{s^2 + 3s + 2}$$

$$\frac{2}{s+2} \bigg| \frac{3}{s+1}$$

$$\frac{s+3}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{2}{s+1} - \frac{1}{s+2}$$

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2M

$$[SI - A] = \begin{bmatrix} S+3 & -2 \\ 1 & S \end{bmatrix}$$

Q7.  $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

$$\Phi(t) = L^{-1} \{ [SI - A]^{-1} \}$$

$$[SI - A]^{-1} = \frac{\begin{bmatrix} S+3 & -2 \\ 1 & S \end{bmatrix}}{S(S+3)+2}$$

$$[SI - A]^{-1} = \frac{\begin{bmatrix} S+3 & -2 \\ 1 & S \end{bmatrix}}{S^2 + 3S + 2}$$

$$\frac{2}{2} \mid \frac{3}{1} \quad \frac{S+3}{(S+1)(S+2)}$$

$$= \frac{A}{S+1} + \frac{B}{S+2} = \frac{2}{S+1} - \frac{1}{S+2}$$

$$S+3 = A(S+2) + B(S+1)$$

$S = -1 \Rightarrow 2 = A$

$S = -2 \Rightarrow 1 = -B \Rightarrow B = -1$

$\Phi(t) =$

$$= \frac{2}{S+2} - \frac{1}{S+1} = 2e^{-2t} - e^{-t}$$

$$= -e^{-t} + 2e^{-2t}$$

A

-B

B=2

$$\frac{S}{(S+2)(S+1)} = \frac{A}{S+2} + \frac{B}{S+1}$$

$S = -2 \Rightarrow -2 = -A$

$A = 2$

$$\frac{-2}{(S+2)(S+1)} = \frac{A}{S+2} + \frac{B}{S+1} = \frac{2}{S+2} - \frac{1}{S+1}$$

$-2 = A(S+1) + B(S+2) = 2e^{-2t} - 2e^{-t}$

$S = -1 \Rightarrow -2 = B$

$A = 2$

$$\frac{1}{(S+2)(S+1)} = \frac{A}{S+2} + \frac{B}{S+1} = \frac{-1}{S+2} + \frac{1}{S+1}$$

$1 = A(S+1) + B(S+2)$

$1 = B \Rightarrow B = 1$

$A = -1$



For a state model :

$$\dot{X} = AX, \text{ where } A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

State transition matrix is:

(a)  $\begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$

(b)  $\begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$

(c)  $\begin{bmatrix} e^{2t} & 0 \\ te^{2t} & e^{2t} \end{bmatrix}$

(d)  $\begin{bmatrix} e^{-2t} & 0 \\ te^{-2t} & e^{-2t} \end{bmatrix}$

UPSC Poly. Lect. 2019

Q.8].  $[sI - A] = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$

$$\varphi(s) = L^{-1} \left\{ \begin{array}{cc} s-1 & 0 \\ 1 & s-1 \end{array} \right\}$$


---


$$\frac{1}{(s-1)^2}$$

$$\varphi(t) = L^{-1} \left\{ \begin{array}{cc} \frac{s-1}{(s-1)^2} & 0 \\ 1 & \frac{s-1}{(s-1)^2} \end{array} \right\}$$

(A)

$$= \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$



For a linear system the state coefficient matrices  $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $C = [1 \ 1]$ ,  $D = 0$

Find the transfer function.

(a)  $\frac{s-3}{s^2-2s+5}$

(b)  $\frac{s-13}{s^2+12s+5}$

(c)  $\frac{s+3}{s^2-2s+9}$

(d)  $\frac{s-30}{5s^2-8s+15}$

CIL MT 2020

$$T.F = [1 \ 1] \begin{bmatrix} s-1 & 2 \\ -2 & s-1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{[1 \ 1] \begin{bmatrix} s-1 \\ -2 \end{bmatrix}}{(s-1)^2 + 4} = \frac{s-3}{s^2-2s+5}$$

Q.9].  $T.F = C [sI-A]^{-1} B.$

$$[sI-A] = \begin{bmatrix} s-1 & -2 \\ 2 & s-1 \end{bmatrix}$$

$$|sI-A| = (s-1)^2 + 4$$

$$\text{adj } sI-A = \begin{bmatrix} s-1 & 2 \\ -2 & s-1 \end{bmatrix}$$



Consider the system  $T(s) = \frac{Y(s)}{U(s)} = \frac{s+3}{s^2+3s+2}$

Obtain the state space representation in the canonical form.

(a)  $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u(t)$

$y(t) = [3 \quad -11] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

(b)  $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 10 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 20 \\ 1 \end{bmatrix} u(t)$

$y(t) = [13 \quad 1] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

2

$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = [3 \quad 1]$

c

(c)  $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

$y(t) = [3 \quad -1] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

(d)  $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u(t)$

$y(t) = [-3 \quad 1] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

CIL MT 2020



The system described by the following state equations-

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; y = [1 \ 1] x$$

$$\dot{X} = Ax + Bu$$
$$y = Cx$$

1. Completely controllable ✓
2. Completely observable ✓

Which of the above statement is/are correct?

- (a) 1 only                      (b) 2 only  
(c) Both 1 and 2              (d) Neither 1 nor 2

RPSC AE 2018

$$Q_c = |B \ AB| \neq 0.$$

$$AB = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \quad Q_c = \begin{vmatrix} 0 & 1 \\ 1 & -3 \end{vmatrix} = -1 \neq 0.$$

$$Q_o = |C^T \ A^T C^T| \neq 0. \quad Q_o = \begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix} = -2 - 2 = -4 \neq 0$$

Q.1]  $A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$      $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$   
 $C = [1 \ 1]$                        $C^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$A^T C^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 - 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

Ⓒ

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0.4 x

### Which of the following statements are correct?

1. The pair  $(AB)$  is controllable implies that the pair  $(A^T B^T)$  is observable.
  2. The pair  $(AB)$  is controllable implies that the pair  $(A^T B^T)$  is unobservable.
  3. The pair  $(AC)$  is observable implies that the pair  $(A^T C^T)$  is controllable.
  4. The pair  $(AC)$  is observable implies that the pair  $(A^T C^T)$  is uncontrollable.
- (a) 1 and 3 only                      (b) 1 and 4 only  
(c) 2 and 3 only                      (d) 2 and 4 only

where :  $A$ ,  $B$  and  $C$  are having their standard meanings.

ESE 2020

Pair of  $AB \Rightarrow$  Controllable.

Pair of  $A^T B^T \Rightarrow$  observable.

Pair of  $(AB) \Rightarrow$  observable.

Pair of  $A^T B^T \Rightarrow$  Controllable.

$$Q_o = |C^T A^T C^T|$$

$$Q_c = |B \ AB|$$

A

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0.7 x

The value of A matrix in  $\dot{X} = AX$  for the system described by  $\ddot{y} + 2\dot{y} + 3y = 0$

(a)  $\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$

(c)  $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(d)  $\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$

TANGEDCO AE 2018

$y'' + 2y' + 3y = 0$   
 $A = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$



The state model of a system is represented as  
 $\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$  and output  $y = [1 \ 0] X$ .  
 Its transfer function will be  
 (a)  $\frac{1}{s^2 + 3s + 3}$  (b)  $\frac{2}{s^2 + 3s + 2}$   
 (c)  $\frac{3}{s^2 + 3s + 3}$  (d)  $\frac{1}{s^2 + 3s + 2}$   
 UKPSC AE 2012, Paper-I

Q.4].  $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$   $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = [1 \ 0]$

T.F =  $C [sI - A]^{-1} B$

$[sI - A] = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$

(d)

=  $\frac{C \text{adj}[sI - A] B}{|sI - A|}$

=  $\frac{[1 \ 0] \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s(s+3) + 2}$

=  $\frac{[1 \ 0] \begin{bmatrix} 1 \\ s \end{bmatrix}}{s^2 + 3s + 2} = \frac{1}{s^2 + 3s + 2}$



Which of the following systems is completely controllable?

(a)  $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$

(b)  $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$

(c)  $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

(d)  $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} u$

UPPSC AE 2008, Paper-I

(A)  $\Rightarrow A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$Q_c = |B \quad AB| \neq 0$  (A) ✓

$= \begin{vmatrix} 2 & -2 \\ 1 & -2 \end{vmatrix} = -4 + 2 \neq 0$

(B)  $\Rightarrow A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$Q_c = \begin{vmatrix} 2 & -2 \\ 0 & 0 \end{vmatrix} = 0$

(C)  $\Rightarrow A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad Q_c = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  (A)

(D)  $A = \begin{bmatrix} 0 & 0 \\ 2 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad Q_c = \begin{vmatrix} 2 & 0 \\ 2 & 0 \end{vmatrix} = 0$

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0.4 x

Consider a linear time-invariant system whose input  $r(t)$  and output  $y(t)$  are related by the following differential equation:

$\frac{d^2y(t)}{dt^2} + 4y(t) = 6r(t)$

The poles of this system are at

- $+2j, -2j$
- $+2, -2$
- $+4, -4$
- $+4j, -4j$

$\Rightarrow$  State model.  $A = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}$ .

$\frac{d^2y(t)}{dt^2} + 0 \cdot \frac{dy}{dt} + 4y(t) = 6r(t)$ .

GATE 2020, EE

$(A)$

C.E. =  $|sI - A| = \begin{vmatrix} s & -1 \\ 4 & s \end{vmatrix}$

$= s^2 + 4 = 0$

$s^2 = -4$

$s = \pm i\sqrt{4} = \pm i2$

$s = +2i, -2i$



Consider the differential equation  $\ddot{y} + 2\dot{y} + y = u$ , where  $y(0^-) = 0$ ,  $\dot{y}(0^-) = 0$  and  $u$  is a unit step function. The poles of the system are

(a)  $s_1 = -1, s_2 = -1$       (b)  $s_1 = j1, s_2 = -j1$   
 (c)  $s_1 = -1, s_2 = -2$       (d)  $s_1 = 1, s_2 = 2$

UKPSC AE 2012, Paper-I

II

$$s^2 Y(s) + 2sY(s) + Y(s) = U(s).$$

$$Y(s) [s^2 + 2s + 1] = U(s).$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 2s + 1}$$

Poles  $\Rightarrow s^2 + 2s + 1 = 0$   
 $\therefore s = -1, -1$

$$\begin{array}{c|cc} s^2 & 1 & 1 \\ s^1 & 2 & \\ s^0 & 1 & \end{array}$$

I

$$\frac{d^2y}{dt^2} + 2 \cdot \frac{dy}{dt} + y = u.$$

$$|SI - A| = \begin{vmatrix} s & -1 \\ 1 & s+2 \end{vmatrix} = 0.$$

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

$$s(s+2) + 1 = 0.$$

$$s^2 + 2s + 1 = 0$$

$$(s+1)(s+1) = 0$$

$$s = -1, -1$$

$$\begin{array}{c|cc} & 1 & 2 \\ & 1 & 1 \end{array}$$



The transfer function of a system is:

$$T.F = \frac{(s + 1)(s + 3)}{(s + 5)(s + 7)(s + 9)}$$

In the state-space representation of the system, the minimum number of state variables (in integer) necessary is 3

GATE 21/22

$$T.F = \frac{(s+1)(s+3)}{(s^2+12s+35)(s+9)}$$

$$= \frac{(s+1)(s+3)}{3s^3 + 9s^2 + 7s^4 + s^3 + 21s^2 + 143s + 315}$$

$$= \frac{(s+1)(s+3)}{3s^3 + 9s^2 + 7s^4 + s^3 + 21s^2 + 143s + 315}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -315 & -143 & -215 \end{bmatrix} 3 \times 3$$



The state space representation of a first-order system is given as

$$\begin{aligned}\dot{x} &= -x + u \\ y &= x\end{aligned}$$

where,  $x$  is the state variable,  $u$  is the control input and  $y$  is the controlled output. Let  $u = -Kx$  be the control law, where  $K$  is the controller gain. To place a closed-loop pole at  $-2$ , the value of  $K$  is \_\_\_\_\_.

GATE 21, EE

$$\begin{aligned}\dot{x} &= -x - Kx = x[-K-1] + 0u \\ y &= x.\end{aligned}$$

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx.\end{aligned}$$

$$A = -K-1 \quad B = 0 \quad C = 1$$

$$\begin{aligned}\text{C.E} &= |sI - A| = 0 \quad \boxed{s = -2} \\ &= |s - [-K-1]| = 0 \\ &= [s + K + 1] = 0 \\ &= -2 + K + 1 = 0 \\ &\quad \boxed{K = 1}\end{aligned}$$

Full Video Link



0.7 x

A system is described by the state equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mu$$

The state transition matrix of the system is

(a)  $\begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t} \end{bmatrix}$

(b)  $\begin{bmatrix} 0 & e^{2t} \\ e^{2t} & 0 \end{bmatrix}$

(c)  $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$

(d)  $\begin{bmatrix} e^{2t} & 1 \\ 1 & e^{2t} \end{bmatrix}$

BPSC Poly. Lect. 2016

$$\Phi(t) = e^{At} = L^{-1} \{ [SI - A]^{-1} \}$$

$$[SI - A] = \begin{bmatrix} s-2 & 0 \\ 0 & s-2 \end{bmatrix}$$

$$|SI - A| = (s-2)^2$$

$$[SI - A]^{-1} = \frac{1}{(s-2)^2} \begin{bmatrix} s-2 & 0 \\ 0 & s-2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s-2} & 0 \\ 0 & \frac{1}{s-2} \end{bmatrix} \quad \text{(A)}$$
$$= \begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t} \end{bmatrix}$$

Full Video Link



0.5 x

Q.33 Let the state-space representation of an LTI system be  $\dot{x}(t) = A x(t) + B u(t)$ ,  $y(t) = C x(t) + d u(t)$  where  $A, B, C$  are matrices,  $d$  is a scalar,  $u(t)$  is the input to the system, and  $y(t)$  is its output. Let  $B = [0 \ 0 \ 1]^T$  and  $d = 0$ . Which one of the following options for  $A$  and  $C$  will ensure that the transfer function of this LTI system is

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1}?$$

- (A)  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$  and  $C = [1 \ 0 \ 0]$
- (B)  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$  and  $C = [1 \ 0 \ 0]$
- (C)  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$  and  $C = [0 \ 0 \ 1]$
- (D)  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$  and  $C = [0 \ 0 \ 1]$

Handwritten notes:

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [1 \ 0 \ 0]$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$$

(A)

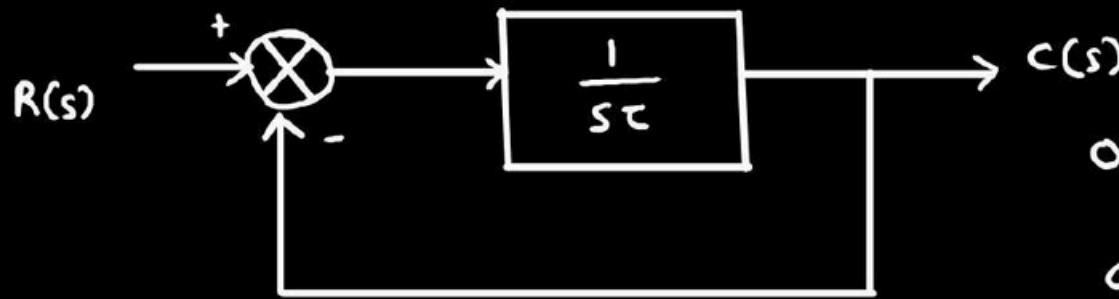
GATE 19, EC (2M)



# Time Response.

\* Total Response = Transient Response + Steady State Response.

\* Time Response of First order Systems:



$$OLTF = \frac{1}{s\tau}$$

$$CLTF = \frac{1}{1+s\tau}$$

\* Type  $\Rightarrow$  Origin order  $\Rightarrow$  Total no. of poles.

\* Order and Type  $\begin{cases} OLTF \checkmark \\ CLTF \times \end{cases}$

\* Order also depends on memory circuit elements.

OLTF =  $\frac{1}{s}$   
 $s=0$   
 $s=0, 0, 1$   
 $\downarrow$   
2 poles

OLTF =  $\frac{s+2}{s^2(s+1)}$

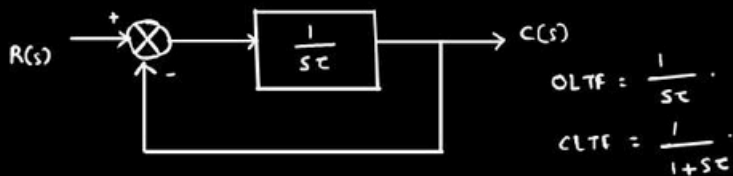
Full Video Link



## Time Response.

\* Total Response = Transient Response + Steady State Response.

\* Time Response of First order System:



$$OLTF = \frac{1}{s\tau}$$

$$CLTF = \frac{1}{1+s\tau}$$

$$OLTF = \frac{1}{s\tau}$$

$s=0$  TYPE 1 system

$s = 0, 0, -1, -3$   
 $\downarrow$   
 2 poles  $\Rightarrow$  TYPE 2.  
 $\Rightarrow$  4<sup>th</sup> order.

$$OLTF = \frac{N}{D}$$

$$CLTF = \frac{N}{D+N-N}$$

$$CLTF = \frac{s+2}{s^2(s+1)(s+3)+s+2}$$

$$OLTF = \frac{s+2}{s^2(s+1)(s+3)+s+2} \parallel$$

\* Type  $\Rightarrow$  Origin order  $\Rightarrow$  Total no. of poles.

$$\Rightarrow OLTF = \frac{s+2}{s^2(s+1)(s+3)}$$

\* Order and Type  $\begin{cases} OLTF \checkmark \\ CLTF \times \end{cases}$

\* Order also depends on memory circuit elements.

Input $r(t)$	Output $c(t)$	Error $e(t)$
Impulse Response $\delta(t)$	$c(t) = \frac{1}{\tau} e^{-t/\tau}$	0
Unit Step $u(t)$	$c(t) = 1 - e^{-t/\tau}$	$e^{-t/\tau}$
Unit Ramp $r(t)$	$c(t) = t - \tau(1 - e^{-t/\tau})$	$\tau(1 - e^{-t/\tau})$

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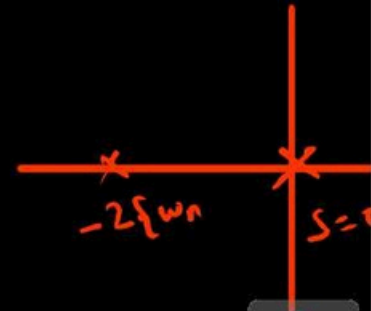
\*

Input $r(t)$	Output $c(t)$	Error $e(t)$
Impulse Response $\delta(t)$	$c(t) = \frac{1}{\tau} e^{-t/\tau}$	0
Unit Step $u(t)$	$c(t) = 1 - e^{-t/\tau}$	$e^{-t/\tau}$
Unit Ramp $r(t)$	$c(t) = t - \tau(1 - e^{-t/\tau})$	$\tau(1 - e^{-t/\tau})$
Parabola $p(t)$	$c(t) = \frac{At^2}{2} + Bt + C + D e^{-t/\tau}$	$Bt + C + D e^{-t/\tau}$

\* Time Response of 2<sup>nd</sup> order System:

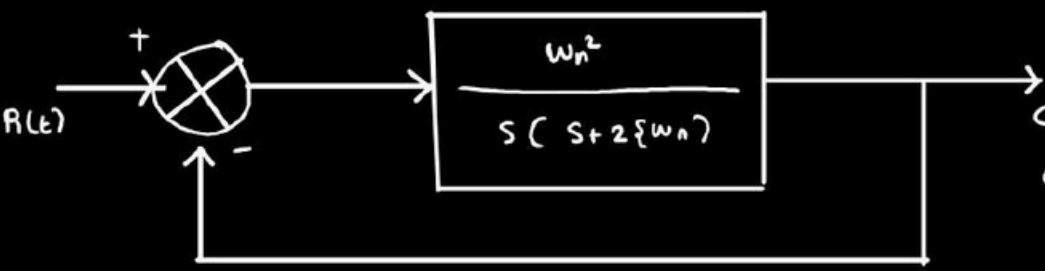
→ OLTF =  $\omega$

$s = 0$   
 $s = -2\zeta\omega_n$   
 Type = 1  
 order = 2



$Y(t)$		
Parabola	$C(t) = \frac{At^2}{2} + Bt + C + D e^{-t/\tau}$	$Bt + C + D e^{-t/\tau}$
$P(t)$		

\* Time Response of 2<sup>nd</sup> order System:



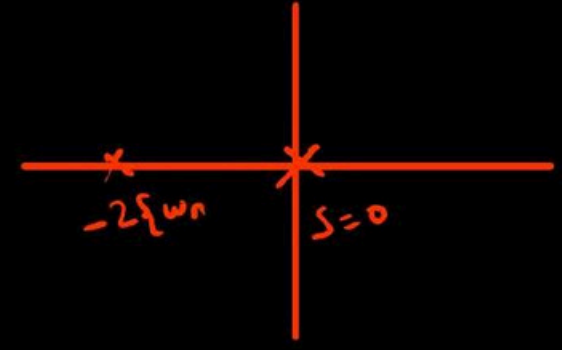
→ OLTF =  $\omega$

$$OLTF = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s}$$

$$CLTF = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$= \frac{N}{D}$   
 $= \frac{N}{D+N}$

Type = 1  
order = 2  
 $s = 0$   
 $s = -2\xi\omega_n$   
Type = 1



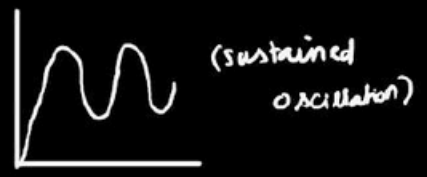
\* Output value depends on {

i].  $\xi = 0$  undamped system

$$s = \pm j\omega_n$$

$$\tau = \frac{-1}{\text{real part of dominant pole}}$$

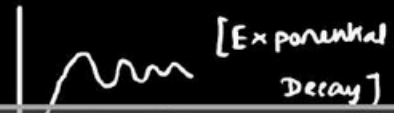
$$\tau = \frac{-1}{0} = \infty$$



ii].  $0 < \xi < 1$  under damped system.

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$\tau = \frac{-1}{-\xi\omega_n} = \frac{1}{\xi\omega_n}$$



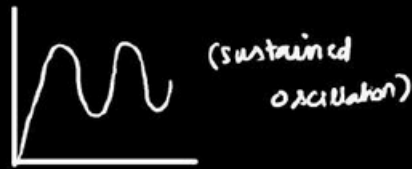
Full Video Link



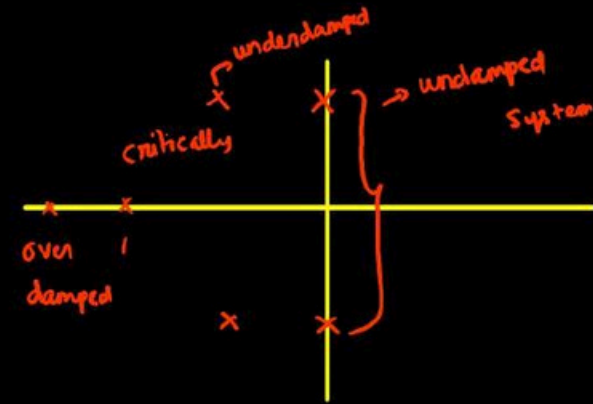
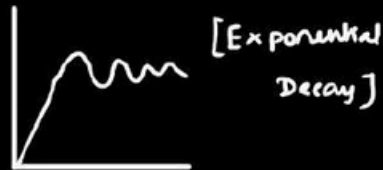
$$\mathcal{LTF} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{D+N}$$

\* Output value depends on  $\zeta$

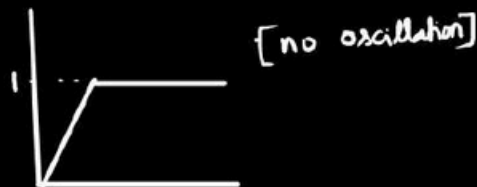
i].  $\zeta = 0$  undamped system  
 $s = \pm j\omega_n$   $\tau = \frac{-1}{\text{real part of dominant pole}}$   
 $\tau = \frac{-1}{0} = \infty$



ii].  $0 < \zeta < 1$  under damped system.  
 $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$   $\tau = \frac{-1}{-\zeta\omega_n} = \frac{1}{\zeta\omega_n}$



iii].  $\zeta = 1$  critical damped system.  
 $s = -\omega_n, -\omega_n$   $\tau = \frac{1}{\omega_n}$



iv].  $\zeta > 1$  over damped system.  
 $s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$   $\tau = \frac{-1}{-\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}}$



v].  $\zeta < 0 \Rightarrow$  unstable system.



Time Constant:

$\xi = 0$

$\xi > 1$

$0 < \xi < 1$

$\xi = 1$

Undamped > overdamped > underdamped > critical.

Time Domain Specifications:

\* Delay Time:  $t_d = \frac{1 + 0.7\xi}{\omega_n}$

\* Rise Time:  $t_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - \cos^{-1}\xi}{\omega_n \sqrt{1 - \xi^2}}$

\* Peak Time:  $t_p = \frac{n\pi}{\omega_d}$   $n = 1, 3, 5$  overshoot.  
 $= 2, 4, 6$  undershoot.

2<sup>nd</sup> overshoot  $\rightarrow n = 3$

2<sup>nd</sup> undershoot  $\rightarrow n = 4$ .

2<sup>nd</sup> overshoot  $\rightarrow n = 6$ .

$t_p = \frac{3\pi}{\omega_d}$

Full Video Link



\* Peak time:  $t_p = \frac{\pi}{\omega_d} = 2, 4, 6$  underdamped.

2<sup>nd</sup> overshoot  $\rightarrow n = 3$       2<sup>nd</sup> undershoot  $\rightarrow n = 4$ .

3<sup>rd</sup> overshoot  $\rightarrow n = 5$       3<sup>rd</sup> undershoot  $\Rightarrow n = 6$ .

$$t_p = \frac{3\pi}{\omega_d}$$

\* Peak overshoot:  $m_p = e^{-\frac{\zeta n \pi}{\sqrt{1-\zeta^2}}}$

\* Maximum transient peak:  $1 + m_p$ .

\* Settling Time:

$$0\% \Rightarrow t_s = 5\tau$$

$$\pm 2\% \Rightarrow t_s = 4\tau$$

$$\pm 5\% \Rightarrow t_s = 3\tau.$$

+ Miscellaneous Topics:

Full Video Link



2.0 x

\* Miscellaneous Topic:

i]. 1<sup>st</sup> order  $\Rightarrow$  RL, RC. RL circuit  $\tau = L/R$

RC circuit  $\tau = RC$

ii]. 2<sup>nd</sup> order  $\Rightarrow$  RLC  $\omega_n^2 = \frac{1}{LC}$   $2\zeta\omega_n = \frac{R}{L}$

iii]. Damping  $= \frac{1}{\sqrt{\text{gain}}}$  \*  $\zeta = \frac{\text{Actual Damping}}{\text{Critical Damping}}$

v]. Bandwidth  $= \frac{0.35}{t_r}$

Steady State Error.

\*  $E(s) = \frac{R(s)}{1+G(s)} \Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$

\*  $E_{ss} = \lim_{s \rightarrow 0} s \cdot E(s) = \lim_{s \rightarrow 0} s \cdot \frac{R(s)}{1+G(s)}$

\* Step I/p  $\Rightarrow Au(t) \Rightarrow E_{ss} = \frac{A}{1+k_p}$   $k_p = \lim_{s \rightarrow 0} G(s)$   $k_p \rightarrow$  proportional

\* Ramp I/p  $\Rightarrow Atu(t)$

Series RLC circuit

CLTF =  $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$   $\zeta = \frac{R}{2} \sqrt{\frac{C}{L}}$   
 $\omega_n = \frac{1}{\sqrt{LC}}$   
 $\frac{R}{L}$

Q.]. The B.W of a low pass RC circuit is 1 kHz. What is the value of  $t_r$  of output for  $u(t)$ ?

BW = 1 kHz.  $t_r = ?$

B.W =  $\frac{0.35}{t_r} \Rightarrow t_r = \frac{0.35}{1 \times 10^3} = 0.35 \text{ mSec.}$

$5u(t)$   
 $5tu(t)$   
 $5t^2u(t)$

$\frac{10}{2}t^2u(t)$

Handwritten notes and icons at the bottom of the page, including a calculator icon and some scribbles.



## Steady State Error.

$$* E(s) = \frac{R(s)}{1+G(s)} \Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$

$$B.W = \frac{0.35}{t_r} \Rightarrow t_r = \frac{0.35}{1 \times 10^3} = 0.35 \text{ m Sec.}$$

$$* E_{ss} = \lim_{s \rightarrow 0} s \cdot E(s) = \lim_{s \rightarrow 0} s \cdot \frac{R(s)}{1+G(s)}$$

5u(t).  
5tu(t).  
5t<sup>2</sup>u(t).

$$* \text{Step I/p} \Rightarrow Au(t) \Rightarrow E_{ss} = \frac{A}{1+k_p}$$

$$k_p = \lim_{s \rightarrow 0} G(s) \cdot \frac{10 \cdot t^2 u(t)}{2} \quad k_p \rightarrow \text{proportional}$$

$$* \text{Ramp I/p} \Rightarrow Atu(t) \Rightarrow E_{ss} = \frac{A}{k_v}$$

$$k_v = \lim_{s \rightarrow 0} s \cdot G(s) \quad k_v \rightarrow \text{velocity}$$

$$* \text{Parabola I/p} \Rightarrow \frac{A}{2} t^2 u(t)$$

$$\Rightarrow E_{ss} = \frac{A}{k_a}$$

$$k_a = \lim_{s \rightarrow 0} s^2 \cdot G(s) \quad k_a \rightarrow \text{acceleration constant}$$

Input < Type ;  $E_{ss} = 0$ .

Input = Type ;  $E_{ss} = k_i$  [ $k_i = k_p = k_a = k_v$ ]

Input > Type ;  $E_{ss} = \infty$



Full Video Link



Q.2]. What is the time required to reach 98% of Steady state value ( 2% tolerance band) for the closed loop Transfer

Function  $\frac{2}{(s+10)(s+100)}$  when input is step response?

Sol.  $t_s \Rightarrow t_s = 4\tau$

0%	5 $\tau$
2%	4 $\tau$
5%	3 $\tau$

$\tau = ?$

$$CLTF = \frac{2}{10 \left(1 + \frac{s}{10}\right) 100 \left(1 + \frac{s}{100}\right)}$$

$$= \frac{2}{1000 \left(1 + \frac{s}{10}\right)}$$

$$CLTF = \frac{1}{1+s\tau}$$

$$\tau = \frac{1}{10} = 0.1$$

$t_s = 4\tau = 0.4 \text{ sec}$



The transfer function for a system with input  $r(t) = 1 + e^{-3t}$  and response  $c(t) = e^{-2t}$  is:

- (a)  $(s + 2) / (s + 3)$  ✖
- (b)  $(s + 3) / (s + 2)$  ✖
- (c)  $(s + 2) / (s + 2)(s + 3)$  ✖
- (d)  $s(s + 3) / (s + 2)(2s + 3)$  ✖

LMRC AM 2020

(D)

Q1].

Input =  $r(t) = 1 + e^{-3t}$       T.F =  $\frac{C(s)}{R(s)}$

Output =  $c(t) = e^{-2t}$

$C(s) = \frac{1}{s+2}$

$R(s) = \frac{1}{s} + \frac{1}{s+3}$

$= \frac{s+3+s}{s(s+3)} = \frac{2s+3}{s(s+3)}$

T.F =  $\frac{\frac{1}{s+2}}{\frac{2s+3}{s(s+3)}}$

$= \frac{1}{(s+2)} \times \frac{(s+3)s}{(2s+3)}$



111.  
The value of unit impulse function  $\delta(t)$  for  $t > 0$  is:  $t = 0$

(a) zero ✓  
(b) unity  
(c) infinite  
(d) constant

APPSC Poly. Tech. Lect. 2020

A

Q.2].  $\delta(t) = 1$  for  $t = 0$ .  
otherwise  $\delta(t) = 0$ .



Which of the following transfer function of second order linear time-invariant systems, the under damped system is represented by?

- (a)  $H(s) = \frac{1}{s^2 + 4s + 4}$       (b)  $H(s) = \frac{1}{s^2 + 5s + 4}$   
 (c)  $H(s) = \frac{1}{s^2 + 4.5s + 4}$       (d)  $H(s) = \frac{1}{s^2 + 3s + 4}$

ISRO Scientist/Engineer 2020

Q.3]. 
$$= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

(A).  $2\zeta\omega_n = 4$        $\omega_n^2 = 4$   
 $\omega_n = 2$   
 $\zeta = \frac{2}{2} = 1 \Rightarrow$  critically.

(D).  $\omega_n = 2$   
 $2\zeta\omega_n = 3$   
 $\zeta = \frac{3}{4} = 0.75$

(B)  $\Rightarrow \omega_n^2 = 4$        $2\zeta\omega_n = 5$        $1.25$   
 $\omega_n = 2$        $\zeta = \frac{5}{2 \times 2} = \frac{5}{4}$   
 $\zeta > 1$  overdamped.

(C)  $\Rightarrow \omega_n^2 = 4$        $2\zeta\omega_n = 4.5$        $1.125$   
 $\omega_n = 2$        $\zeta = \frac{4.5}{4}$   
 $\zeta > 1$  overdamped.

$0 < \zeta < 1$  underdamped.



In a control system, the response is critically damped if:

- (a) Damping factor is 0 → undamped.
- (b) Damping factor is  $< 1$  →  $\zeta < 1 \Rightarrow$  underdamped.
- (c) Damping factor is  $> 1$
- (d) Damping factor is 1

BHEL ET 2019

Q.4].  $\zeta = 1$

$\zeta > 1 \Rightarrow$  over damped



If the damping factor of a control system is unity, its response will be:

(a) oscillatory                      (b) un-damped  
(c) under-damped                  (d) critically damped

APPSC AEE 2019  
LMRC AM 2018  
ESE 1992

d

Q.5].  $\zeta = 1 \Rightarrow$  critical



A unity feedback system has the following unit impulse response :

$$c(t) = -te^{-t} + 2e^{-t}, (t \geq 0)$$

The open loop transfer function of the system is:

(a)  $\frac{2s+1}{s^2}$

(b)  $\frac{s+1}{s^2}$

(c)  $\frac{s+1}{(s+2)^2}$

(d)  $\frac{s}{(s+1)^2}$

APTRANSCO AE 2019

A

$$L[t^n] = \frac{n!}{s^{n+1}}$$

$$L[t^1] = \frac{1!}{s^2}$$

$$CLTF = \frac{N}{D+N}$$

$$OLTF = \frac{N}{(D+N) - N}$$

Q6. Input Response = Impulse response =  $1 = R(s)$ .

$$c(t) = -te^{-t} + 2e^{-t}$$

$$c(s) = -\frac{1}{(s+1)^2} + \frac{2}{(s+1)}$$

$$T.F = \frac{c(s)}{R(s)} = \frac{-\frac{1}{(s+1)^2} + \frac{2}{(s+1)}}{1} = \frac{-1 + 2s + 2}{(s+1)^2}$$

$$CLTF = \frac{2s+1}{(s+1)^2} = \frac{2s+1}{s^2+1+2s}$$

$$OLTF = \frac{2s+1}{s^2+2s+1 - (2s+1)}$$

$$= \frac{2s+1}{s^2+2s+1-2s-1}$$



0.4 x

Full Video Link



Step response of a system is  $c(t) = 2(1 - e^{-4t})$ ,  $t > 0$ ,  
Determine impulse response of the same system.

- (a)  $2e^{-4t}$ ,  $t > 0$
- (b)  $8e^{-4t}$ ,  $t > 0$
- (c)  $\frac{1}{2}e^{-4t}$ ,  $t > 0$
- (d)  $4e^{-4t}$ ,  $t > 0$

UPPCL AE 04-11-2019 Shift II

Impulse  
 $= \frac{d}{dt} [2 - 2e^{-4t}]$   
Ramp =  $c(t) =$   
Step =  $\frac{d}{dt} [\text{Ramp}]$ .  
Step = Given.  
Impulse =  $\frac{d}{dt} [\text{Step}]$ .  
 $= 8e^{-4t}$ ,  $t > 0$ .

Q.2]. The response of an initially relaxed system to a

Full Video Link



0.4 x

Q.2]. The response of an initially relaxed system to a unit ramp excitation is  $(t + e^{-t})$ . Its step response will be? ESE 2001.

$$\text{Ramp excitation} = t + e^{-t}.$$

$$\text{Step response} = \frac{d}{dt} [\text{ramp excitation}].$$

$$= \frac{d}{dt} [t + e^{-t}].$$

$$= 1 - e^{-t}.$$

Full Video Link



0.3 x

Transfer function of a second order system is  $G(s) = 100/(s^2+20s+100)$ , and this system is classified as:

- (a) Undamped system
- (b) Over damped system
- (c) Critically damped system
- (d) Under damped system

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©.

Q. 8].  $G(s) = \frac{100}{s^2 + 20s + 100}$

$\omega_n^2 = 100$     $\omega_n = 10$     $2\zeta\omega_n = 20$   
 $\zeta = \frac{10}{10} = 1$

Full Video Link



0.5 x

The time constant of the causal system

represented by  $G(S) = \frac{1}{s+5}$  is

- (a)  $10/\pi$  seconds                      (b) 5 seconds  
(c) 0.2 seconds                            (d)  $\pi/10$  seconds

UPPCL AE 01-01-2019 Shift I

© .

$$G(s) = \frac{1}{s+5}$$

$$= \frac{1}{5 \left[ 1 + \frac{s}{5} \right]}$$

$$G(s) = \frac{1}{1+s\tau}$$

$$\tau = \frac{1}{5} = 0.2 \text{ seconds.}$$

Full Video Link



0.5 x

The causal system represented by

$$G(S) = \frac{9}{s^2 + 6s + 9} \text{ is}$$

- (a) Undamped                      (b) Underdamped  
(c) Critically damped            (d) Overdamped

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©

Q.10].  $\omega_n^2 = 9$        $2\zeta\omega_n = 6$

$\omega_n = 3$        $\zeta = \frac{3}{3} = 1$

Critically

Full Video Link



0.5 x

The steady state error for a ramp input for a unity feedback system with open loop transform function  $k/s(s+1)(s+1)$  is:  
 (Given  $k=2$ )  
 (a) 0 (b) 1  
 (c) Infinity (d) 0.5  
 LMRC AM 2020

Input Ramp =  $A t u(t)$   
 $= 1$   
 (d)

Input Type = 1

System  $G(s) = \frac{2}{s(s+1)(s+1)}$

$s=0$   
 $s=-1, -1$  } order = 3.  
 Type = 1.

$E_{ss} = \frac{1}{k_v} \Rightarrow k_v = \lim_{s \rightarrow 0} s \cdot G(s)$

$= \lim_{s \rightarrow 0} s \cdot \frac{2}{s(s+1)(s+1)}$

$E_{ss} = \frac{1}{2} = 0.5$

$= \frac{2}{1}$



Full Video Link



A unity feedback system is given by  

$$G(s) = \frac{10(s+2)}{s^2(s+5)}$$
  
 For input,  $r(t) = 1 + 2t$ ,  $t > 0$  the steady state error  $e(t)$  is:  
 (a) infinity (b) zero  
 (c) six (d) five  
 DMRC AM 2020

$$G(s) = \frac{10(s+2)}{s^2(s+5)}$$
  
 System type = 2. (b)

Q.3].  
 Input =  $1 + 2t'$   
 $= I_1 + I_2$

$I_1$  : Type = 0. System Type = 2  
 $I_P < \text{System} \Rightarrow E_{ss} = 0$   
 $I_2$  = Type 1 system type 2.  $E_{ss} = 0$ .



A unity feedback control system has forward path transfer function  $G(s) = \frac{K}{s(s+2)}$ . If the design specification is that the steady state error due to unit ramp input is 0.05, the value of gain K will be.

(a) 10                      (b) 80  
 (c) 40                      (d) 20

AP GENCO AE 2012  
 BHEL ET 2019

unit step =  $u(t)t^0 \Rightarrow$  Type 0.

(C)

Q.3.  $G(s) = \frac{k}{s(s+2)}$

$E_{ss} = 0.05$ .

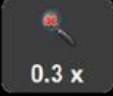
$$E_{ss} = \frac{1}{\frac{k}{2}} = \frac{2}{k} = 0.05$$

$$k = \frac{2}{0.05}$$

$$k = \frac{2 \times 10^{-2}}{5} = \frac{40}{5}$$

Input =  $t^1 u(t) =$  Type = 1      System Type = 1

$$E_{ss} = \frac{1}{k_v} \quad k_v = \lim_{s \rightarrow 0} s \cdot \frac{k}{s(s+2)} = \frac{k}{2}$$



Full Video Link



A unity feedback system has  $G(s) = \frac{k}{s(s+1)}$ .

The steady state error in the output response for unit step input is :

- (a)  $\infty$                       (b) zero  
(c) k                              (d)  $\frac{1}{k}$

APTRANSCO AE 2019

$$E_{ss} = 0 \quad \text{B}$$

I/p < System

Q.4]. Input Response =  $u(t)$  = Type 0.

System = Type 1.

Full Video Link



0.5 x

A unity feedback (negative) system has open loop transfer function  $G(s) = \frac{K}{s(s+2)}$

The closed loop system has steady state unit ramp error of 0.1. The value of gain K should be

- (a) 20                      (b) 30  
(c) 40                      (d) 50

ESE 2019

$$E_{ss} = \frac{1}{\frac{k}{2}} = \frac{2}{k} = 0.1$$

$$\therefore k = \frac{2}{0.1} = 2 \times 10^1 = 20.$$

(A)

Q.5]. System Type = 1  $E_{ss} = 0.1$

I/p = unit ramp =  $t u(t)$  = Type 1.

$$E_{ss} = \frac{1}{k_v} \quad k_v = \lim_{s \rightarrow 0} s \cdot \frac{k}{s(s+2)} = \frac{k}{2}.$$

Full Video Link



0.4 x

A system is having TF =  $\frac{25}{s^2 + 8s + 25}$ . What is the time taken to reach maximum peak overshoot for a step input?

(a)  $\pi/5$                       (b)  $\pi/3$   
 (c)  $\pi/25$                       (d) None of the above

ISRO Scientist/Engineer 2019

~~MP = e<sup>-ζπ/√(1-ζ²)</sup>~~  
~~= e<sup>-0.8π/√(1-0.64)</sup>~~  
~~= e<sup>-0.8π/0.6</sup>~~

~~2ζω<sub>n</sub> = 8~~  
~~ω<sub>n</sub> = 5~~  
~~ζ = 8/10 = 0.8~~

Q.6.  $t_p = \frac{n\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$

$\omega_n = 5$        $2\zeta\omega_n = 8$

$\zeta = \frac{8}{10} = 0.8$

(B)  $= \frac{\pi}{5\sqrt{1-(0.8)^2}} = \frac{\pi}{5\sqrt{0.36}} = \frac{\pi}{5 \times 0.6} = \frac{\pi}{3}$



Full Video Link



Which one of the following transfer function exhibit least steady error for ramp input?  
 (a)  $\frac{9}{s^2 + 2s + 9}$  (b)  $\frac{18}{s^2 + 2s + 18}$   
 (c)  $\frac{36}{s^2 + 2s + 36}$  (d)  $\frac{16}{s^2 + 2s + 16}$   
 UPPCL AE 18-05-2016

(C)

(B) . OLTF =  $G(s) = \frac{18}{s^2 + 2s} = \frac{18}{s[s+2]}$   
 $E_{ss} = \frac{1}{k_v}$   $k_v = \lim_{s \rightarrow 0} s \cdot \frac{18}{s[s+2]} = \frac{18}{2}$

Q.7. A). CLTF =  $\frac{9}{s^2 + 2s + 9} = \frac{9}{s^2 + 2s + 9 - 9}$

$G(s) = \frac{9}{s^2 + 2s} = \frac{9}{s(s+2)}$  . I/p type = 1  
 System = 1.

$E_{ss} = \frac{1}{9} \text{--- (B)}$

$k_v = \lim_{s \rightarrow 0} s \cdot \frac{9}{s(s+2)} = \frac{9}{2}$

$E_{ss} = \frac{1}{k_v} = \frac{2}{9} \text{--- (A)}$

$E_{ss} =$

(D)  $\Rightarrow$  OLTF =  $\frac{16}{s(s+2)}$

$E_{ss} = \frac{1}{k_v} = \frac{1}{2}$

(A)  $\Rightarrow \frac{2}{9}$



Which one of the following transfer function exhibit least steady error for ramp input?

(a)  $\frac{9}{s^2+2s+9}$  (b)  $\frac{18}{s^2+2s+18}$   
 (c)  $\frac{36}{s^2+2s+36}$  (d)  $\frac{16}{s^2+2s+16}$   
 UPPCL AE 18-05-2016

(C)

Q.7. A]. CLTF =  $\frac{9}{s^2+2s+9} = \frac{9}{s^2+2s+2-9}$   
 $G(s) = \frac{9}{s^2+2s} = \frac{9}{s(s+2)}$  I/p type = 1  
 System = 1.

$k_v = \lim_{s \rightarrow 0} s \cdot \frac{9}{s(s+2)} = \frac{9}{2}$

$E_{ss} = \frac{1}{k_v} = \frac{2}{9}$  (A)

(B) . OLTF =  $G(s) = \frac{18}{s^2+2s} = \frac{18}{s[s+2]}$   
 $E_{ss} = \frac{1}{k_v}$   $k_v = \lim_{s \rightarrow 0} s \cdot \frac{18}{s[s+2]} = \frac{18}{2}$

$E_{ss} = \frac{1}{9} = \frac{2}{9}$  (B)

(C) .  $G(s) = \frac{36}{s(s+2)}$

$E_{ss} = \frac{1}{k_v}$   $k_v = \lim_{s \rightarrow 0} s \cdot \frac{36}{s(s+2)} = \frac{36}{2} = 18$   
 $E_{ss} = \frac{1}{18}$  (C)

(D)  $\Rightarrow$  OLTF =  $\frac{16}{s(s+2)}$

$E_{ss} = \frac{1}{k_v} = \frac{1}{8}$  (D)

$k_v = \lim_{s \rightarrow 0} s \cdot \frac{16}{s(s+2)} = \frac{16}{2}$

(A)  $\Rightarrow \frac{2}{9}$   $\downarrow$   
 (B)  $\Rightarrow \frac{1}{9}$   $\downarrow$  second.  
 (C)  $\Rightarrow \frac{1}{18}$   $\rightarrow$  First.  
 (D)  $\Rightarrow \frac{1}{8}$



Full Video Link



0.2 x

A unity- feedback system has the forward path transfer function  $G(s)$ . The steady state error is zero if

- (a)  $G(s)$  is Type-1 and input is unit-ramp.  $\rightarrow \times$
- (b)  $G(s)$  is Type-0 and input is unit-step.
- (c)  $G(s)$  is Type-1 and input is unit-step.
- (d)  $G(s)$  is Type-0 and input is unit-ramp.

UPPCL AE 01-01-2019 Shift II

Input I/p  $< \infty$  Type  $\Rightarrow E_{ss} = 0$ .

Input Type = System Type.

Input Type 0 = System 0

I/p = 0  $< 1 \Rightarrow E_{ss} = \infty$ .

I/p  $> 0 \Rightarrow E_{ss} = 0$ .

Q.8].

(C)

Full Video Link



0.4 x

For the causal system,  $G(s) = \frac{4}{s^2 + 5s + 4}$  the percentage overshoot in the output for a unit step input is

(a) 10% (b) No overshoot  
 (c) 16.3% (d) 5%

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Q.9].

$\omega_n^2 = 4 \quad \omega_n = 2 \quad 2\zeta\omega_n = 5$

$\zeta = 1.25$

$\zeta = \frac{5}{4} = 1.25$

$\zeta > 1 \Rightarrow$  overdamped system.

if  $\zeta = 1 \quad -\pi / \sqrt{1-\zeta^2}$

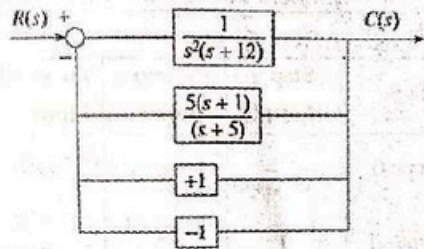
$MP = e^{-\pi / \sqrt{1-\zeta^2}}$   
 $= e^{-\pi / \sqrt{0}} = e^{-\infty} = 0$

No overshoot.

$MP = e^{-1.25\pi / \sqrt{1-1.25^2}}$   
 $= e^{-1.25\pi / \sqrt{-0.5625}} = X$



In the system shown below, what is steady state error in unit ramp response?



- (a) 5/4
- (c) 3/5

- (b) 4/5
- (d) 5/5

ISRO Scientist/Engineer 2018

~~(A)~~

(B) ✓

$$CLTF = \frac{s^2(s+12)}{s^2(s+12)}$$

$$1 + \frac{1}{s^2(s+12)} \times \frac{5(s+1)}{(s+5)} - \frac{1}{s^2(s+12)}$$

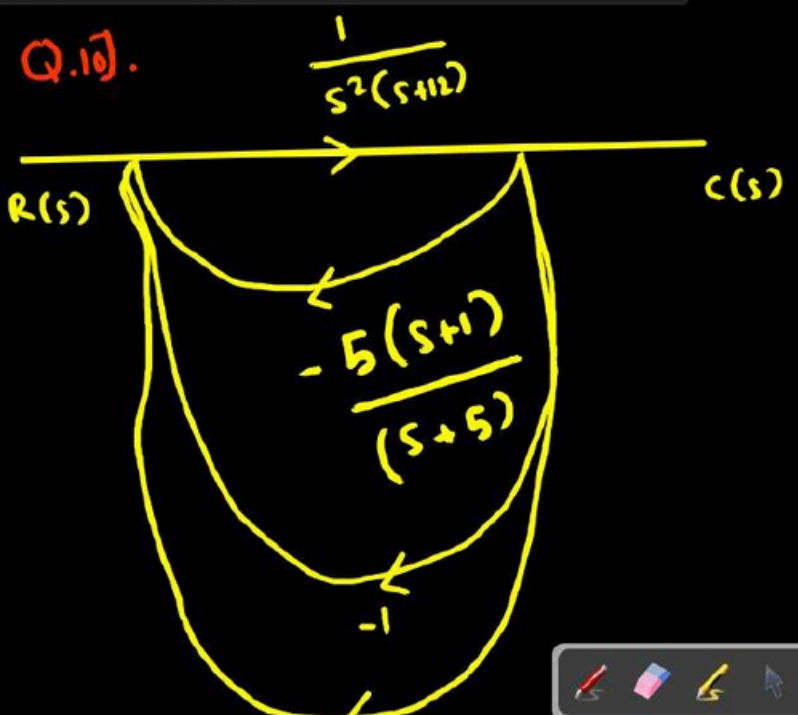
$$CLTF = \frac{1}{s^2(s+12)}$$

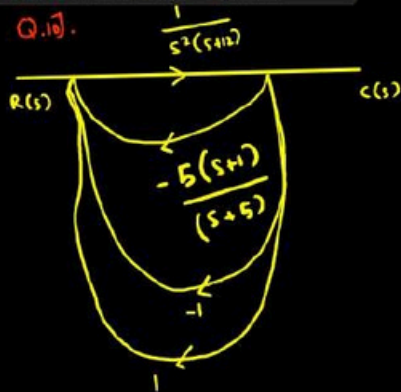
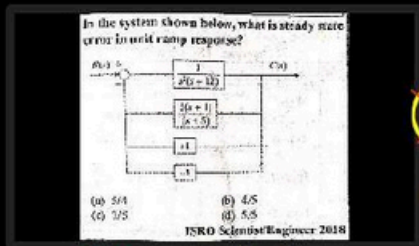
$$\frac{s^2(s+12)(s+5) + 5(s+1)}{s^2(s+12)(s+5)}$$

$$= CLTF =$$

$$CLTF =$$

OLTF





$$CLTF = \frac{1}{s^2(s+12)}$$

$$1 + \frac{1}{s^2(s+12)} \times \frac{5(s+1)}{(s+5)} = \frac{1}{s^2(s+12)} + \frac{1}{s^2(s+12)}$$

~~(A)~~  
(B) ✓

$$CLTF = \frac{1}{s^2(s+12)}$$

$$= CLTF = \frac{1}{s^2(s+12)} \times \frac{s^2(s+12)(s+5)}{s^2(s+12)(s+5) + 5(s+1)}$$

$$CLTF = \frac{s+5}{s^2(s+12)(s+5) + 5s + 5}$$

$$OLTF = \frac{s+5}{s^2(s+12)(s+5) + 4s}$$

$$= \frac{s+5}{s [s(s+12)(s+5) + 4]}$$

$$E_{ss} = \frac{1}{k_v}$$

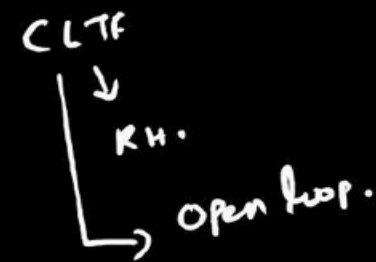
$$k_v = \lim_{s \rightarrow 0} s \cdot \frac{s+5}{s [s(s+12)(s+5) + 4]} = \frac{5}{4}$$

$$E_{ss} = \frac{1}{5/4} = \frac{4}{5}$$



## Root Locus Technique.

\* Purpose of Root Locus is to find the close loop system Stability through open loop systems.



\* At  $k=0$ ; CLP = OLP.

$k=\infty$ ; CLP = OLZ.  
(Loc)

\* No. of RL branches depend on no. of poles and zeros.

if poles are dominated ( $P > Z$ ) no. of RL branches = no. of poles

if zeros are dominated ( $Z > P$ ) no. of RL branches = no. of zeros.

\* -ve feedback system  $\Rightarrow$  Magnitude  $|G(s)H(s)| = 1$

Angle  $\angle G(s)H(s) = 180^\circ$ .

\* +ve feedback system  $\Rightarrow$  Magnitude  $|G(s)H(s)| = 1$

Angle  $\angle G(s)H(s) = 0^\circ$ .

$$T.F = \frac{G(s)}{|1 - G(s)H(s)|} = 0.$$

\* centroid

Full Video Link



# \* Centroid  $\sigma = \frac{\sum p(\text{real}) - \sum z(\text{real})}{p-z} //$

(Intersection point of the asymptote to the real axis)

\* no. of asymptote =  $p-z$ .

Proper T.  $\Rightarrow$  no. of poles = no. of zeros.

\* proper T.F no. of asymptote = 0.

\* If  $p > z$  then it gives direction of zeros.

If  $p < z$  then it gives direction of poles.

$p = 0, -1, 2$   
 $z = 0, 1, \infty$   $p > z$

\* Asymptotic Angle  $\theta = \frac{(2q+1)180^\circ}{p-z}$   $q = 0, 1, 2 \dots (p-z-1)$

\* Angle of Departure  $\phi_d = 180 - \phi$

$\phi = \sum \phi_{(\text{poles})} - \sum \phi_{(\text{zeros})}$

\* Angle of Arrival  $\phi_a = 180 - \phi$



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\* Angle of Departure  $\phi_d = 180 - \phi$

$$\phi = \sum \phi_{(\text{poles})} - \sum \phi_{(\text{zeros})}$$

\* Angle of Arrival  $\phi_A = 180 - \phi$

$$\phi = \sum \phi_{(\text{zeros})} - \sum \phi_{(\text{poles})}$$

\* Root Locus plot intersects the imaginary axis  $\Rightarrow$  Concept of Stability

$\hookrightarrow$  Marginally

To find  $k$  value.

$\omega$   $\odot$ .

Full Video Link



2.0 x

A unity feedback system has open loop transfer function  $G(s) = \frac{K(s+1.1)(s+2.2)}{s(s+3.3)(s+4.4)}$ . For  $K = 0$  the closed-loop poles are :

- (a) All real and distinct
- (b) One real and two complex conjugate
- (c) All real and repeated
- (d) Complex and non repeated

APTRANSCO AE 2017

$\frac{N}{D}$   
 $CLTF = \frac{N}{D+N}$   
A

Q.1]

$$CLTF = \frac{K(s+1.1)(s+2.2)}{s(s+3.3)(s+4.4) + K(s+1.1)(s+2.2)}$$

$K = 0$

$$CLTF = \frac{0}{s(s+3.3)(s+4.4)} = 0$$

$$s(s+3.3)(s+4.4) = 0$$

$$s = 0 \quad s = -3.3$$

$$s = -4.4$$



The root locus of a unity feedback system with the angle made by asymptotes are :

- (a)  $20^\circ, 220^\circ$  ~~x~~      ~~x~~ (b)  $35^\circ, 250^\circ$   
 (c)  $85^\circ, 120^\circ$       (d)  $90^\circ, 270^\circ$

CIL MT 2020

Q.2].  $\theta_A = \frac{(2q+1)180^\circ}{p-z}$        $q = 0, 1, 2, \dots, p-z-1$

$p-z = 1$        $q = 0$

$\theta_A = \frac{(2(0)+1)180^\circ}{1}$

$= 180^\circ$

(D)

$p-z = 3$        $q = 0, 1, 2$

$\theta_{A1} = \frac{2(0)+1}{3} \times 180^\circ = 60^\circ$

$\theta_{A2} = \frac{2(1)+1}{3} \times 180^\circ = 180^\circ$

$p-z = 2$        $q =$

$\theta_{A1} = \frac{(2(0)+1)180^\circ}{2} = 90^\circ$

$\theta_{A2} = \frac{(2(1)+1)180^\circ}{2} = 270^\circ$

Full Video Link



0.5 x

The open loop transfer function of any unity feedback control system is given by  $G(s) = \frac{k(s+2)}{s(s^2+2s+2)}$ . The centroid and angles of asymptotes of the root locus are respectively

(a) zero and  $+90^\circ, -90^\circ$   
 (b)  $-\frac{2}{3}$  and  $+60^\circ, -60^\circ$   
 (c) zero and  $+120^\circ, -120^\circ$   
 (d)  $-\frac{2}{3}$  and  $-90^\circ, +90^\circ$

UKPSC AE 2007, Paper-I

Zeros  $\Rightarrow s = -2 \cdot = 1$

poles  $\Rightarrow s = 0, s = -1+i, s = -1-i \cdot = 3$

$$\sigma = \frac{\sum P(\text{real}) - \sum Z(\text{real})}{P-Z} = \frac{0 - 1 - 1 - (-2)}{3-1} = \frac{-2+2}{2} = 0$$

$P-Z = 3-1 = 2$

$q = 0, 1$

$\theta_{\Delta} = \frac{2(0)+1}{2} = 90^\circ$

$\theta_{\Delta 2} = \frac{2(1)+1}{2} \times \frac{180}{q} = 270^\circ$

0.3]

$G(s) = \frac{k(s+2)}{s(s^2+2s+2)}$

$s = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm i\sqrt{4}}{2} = -1 \pm i$

$90^\circ - 270^\circ$

$s+(-1+i), s+(-1-i)$

(A)

$\pm 180^\circ$



Full Video Link



The open loop transfer function of any unity feedback control system is given by  $G(s) = \frac{k(s+2)}{s(s^2+2s+2)}$ . The centroid and angles of asymptotes of the root locus are respectively

(a) zero and  $+90^\circ, -90^\circ$   
 (b)  $\frac{2}{3}$  and  $+60^\circ, -60^\circ$   
 (c) zero and  $+120^\circ, -120^\circ$   
 (d)  $\frac{2}{3}$  and  $-90^\circ, +90^\circ$

UKFSC AE 2007, Paper-I

0.3)

$$G(s) = \frac{k(s+2)}{s(s^2+2s+2)}$$

$$s = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm i2}{2} = -1 \pm i$$

$s + (-1+i), s + (-1-i)$

Zeros  $\Rightarrow s = -2 \Rightarrow 1$

poles  $\Rightarrow s = 0, s = -1+i, s = -1-i \Rightarrow 3$

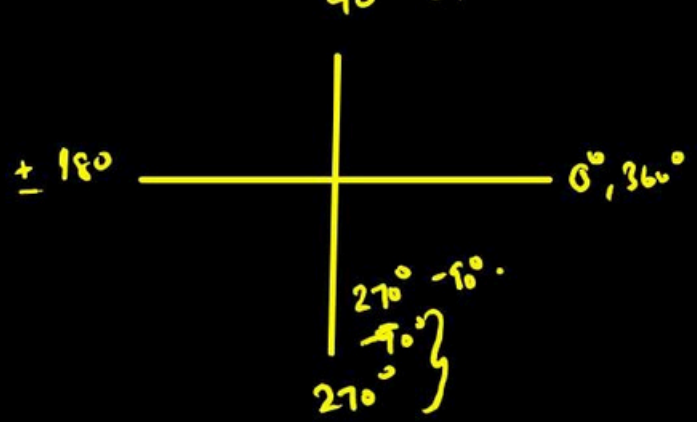
$$\sigma = \frac{\sum p(\text{real}) - \sum z(\text{real})}{p-z} = \frac{0 - 1 - 1 - (-2)}{3-1} = \frac{-2+2}{2} = 0$$

$$p-z = 3-1 = 2$$

$$q = 0, 1$$

$$\theta_{\Delta 1} = \frac{2(0)+1}{2} \times 180^\circ = 90^\circ$$

$$\theta_{\Delta 2} = \frac{2(1)+1}{2} \times 180^\circ = 270^\circ = -90^\circ$$



(A)



0.3 x

Full Video Link



Consider a feedback control system:

The root locus of the system has which of the following characteristics?

- (a) Root loci exist on the negative real axis between  $-3$  and  $-\infty$ . When  $K \rightarrow \infty$ , the root loci terminate at  $s = -2$ , and  $s = -3$ .
- (b) Root loci exist on the negative real axis between  $0$  and  $-1$  between  $-2$  and  $-3$ . When  $K \rightarrow \infty$ , two roots are going to  $-\infty$ .
- (c) Root loci exist on the negative real axis between  $0$  and  $-1$  and between  $-2$  and  $-3$ . When  $K \rightarrow \infty$ , two roots are going to  $\infty$ .
- (d) Root loci exist on the negative real axis between  $0$  and  $-1$  between  $-2$  and  $-3$ . When  $K \rightarrow \infty$ , the root loci terminate at  $s = -2$ , and  $s = -3$ .

UPPCL AE 04-11-2019 Shift II

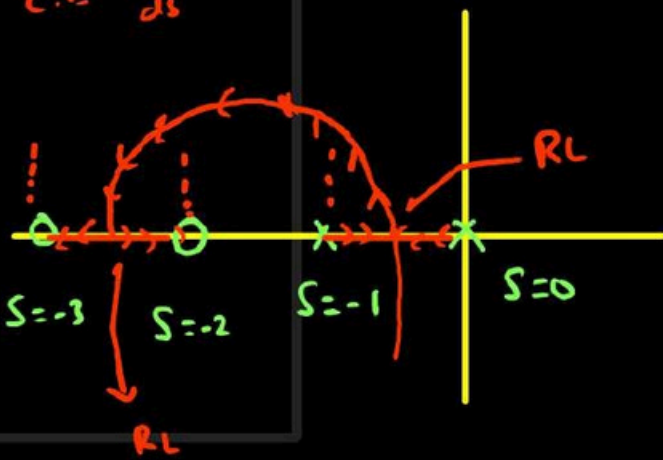
Q.47.  $P = 2$   $Z = 2$  asymptotes =  $P - Z = 0$ .

$G(s) = \frac{K(s+2)(s+3)}{s(s+1)}$

C.E. =  $\frac{dK}{ds} = 0$

$s = 0, s = -1$

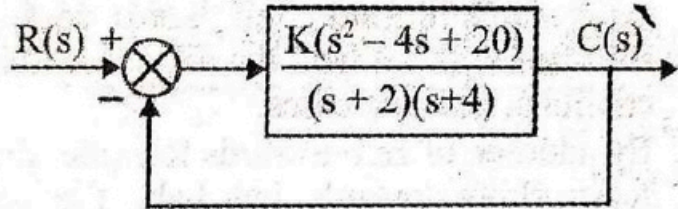
$s = -2, s = -3$



Break away  
Break in



From the given system, determine the number of loci, starting points, ending points and the number of asymptotes.



- (a) 1, (2,4); (2+j4, 2-j4); 0 respectively
- (b) 1, (-2, -4); (2+j4, 2-j4); 0 respectively
- (c) 2, (-2, -4); (2+j4, 2-j4); 0 respectively
- (d) 2, (2, 4); (2+j4, 2-j4); 0 respectively

OMC Deputy Manager 2019

$$s = \frac{4 \pm \sqrt{16 - 80}}{2}$$

$$= \frac{4 \pm \sqrt{-64}}{2}$$

$$= \frac{4 \pm i8}{2} = 2 \pm i4$$

$$\frac{8 \pm 0}{16} = \frac{1}{2}$$

Poles  $\Rightarrow s = -2$   
Zeros  $\Rightarrow s = 2 \pm i4$

X  
X  
X  
X  
X

Q5

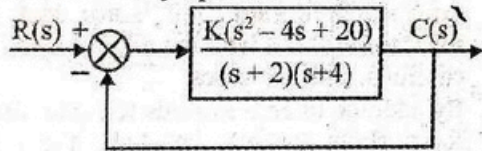
$$G(s) = \frac{K(s^2 - 4s + 20)}{(s+2)(s+4)} \Rightarrow 2$$

No. of RL Branch = no. of poles = no. of zeros = 2

(C)



From the given system, determine the number of loci, starting points, ending points and the number of asymptotes.



- (a) 1, (2,4); (2+j4, 2-j4); 0 respectively
- (b) 1, (-2, -4); (2+j4, 2-j4); 0 respectively
- (c) 2, (-2, -4); (2+j4, 2-j4); 0 respectively
- (d) 2, (2, 4); (2+j4, 2-j4); 0 respectively

OMC Deputy Manager 2019

$$s = \frac{4 \pm \sqrt{16 - 80}}{2}$$

$$= \frac{4 \pm \sqrt{-64}}{2}$$

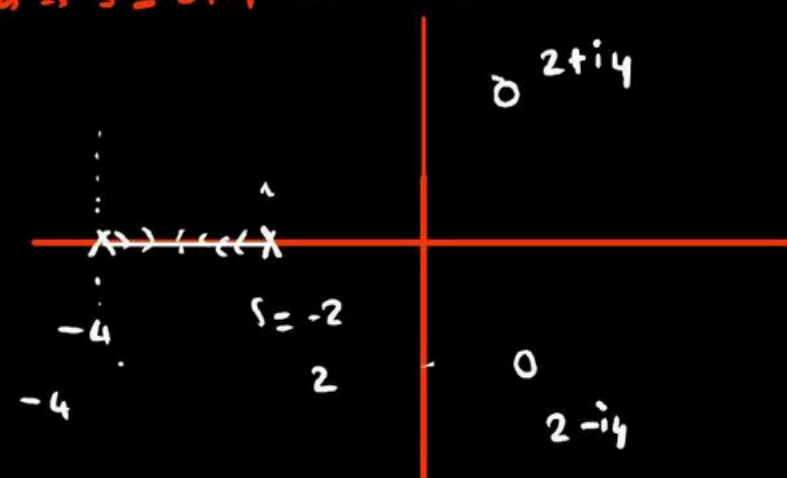
$$= \frac{4 \pm i 8}{2} = 2 \pm i 4$$

(c)

$$\frac{8 \pm 0}{16} = \frac{84}{16}$$

Poles  $\Rightarrow s = -2, s = -4$

Zeros  $\Rightarrow s = 2 + i 4, s = 2 - i 4$



Q.5)  $G(s) = \frac{K(s^2 - 4s + 20)}{(s+2)(s+4)}$

No. of RL Branch = no. of poles = no. of zeros = 2

# no. of asymptotes = 2 - 2 = 0



The centroid of the root locus with open loop transfer function  $k/(s+3)(s^2+3s+2)$  is:

- (a) -3                      (b) -5  
(c) -1                      (d) -2

LMRC AM 2020

Q.5.

$$G(s) = \frac{k}{(s+3)(s^2+3s+2)}$$

$$= \frac{k}{(s+3)(s+2)(s+1)}$$

$$\begin{array}{r|l} 2 & 3 \\ \hline 2 & 1 \end{array}$$

Poles  $\Rightarrow 3$ .  
Zero  $\Rightarrow 0$ .

$$\sigma = \frac{\sum P(\text{real}) - \sum Z(\text{real})}{P-Z}$$

P-Z

$$= \frac{-3-2-1 - (0)}{3} = \frac{-6}{3} = -2.$$

(d)

$$s = -3, -2, -1$$



0.4 x

Full Video Link

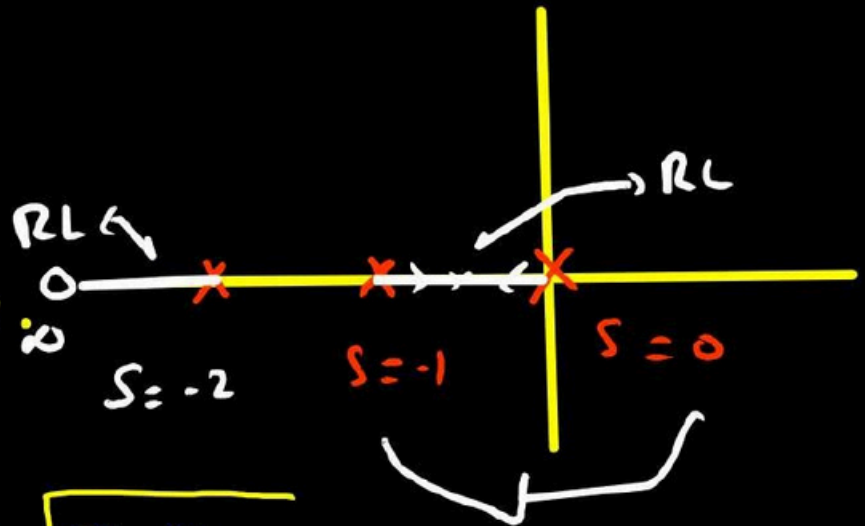


The open loop transfer function of unity feedback control system is  $\frac{k}{s(s+1)(s+2)}$ . The break-away point of its root locus is at  
 (a)  $s = -0.422$  (b)  $s = 1.573$   
 (c)  $s = 0.123$  (d)  $s = -4.62$   
 UKPSC AE 2007, Paper-I

A

$s = -0.423$

$s = -1.577$



Q.7].  $G(s) = \frac{k}{s(s+1)(s+2)}$

C.E =  $s(s+1)(s+2) + k = 0$

$s(s^2 + 3s + 2) = -k$

$s^3 + 3s^2 + 2s = -k$

$3s^2 + 6s + 2 = \frac{-dk}{ds} = 0$

$= \frac{-6 \pm \sqrt{36 - 24}}{6}$

$= \frac{-6 \pm \sqrt{12}}{6}$

$s = \frac{-6 \pm 2\sqrt{3}}{6}$

$$\begin{array}{r} 2 \overline{) 12} \\ 2 \overline{) 6} \\ \hline 3 \end{array}$$

A

$s = \frac{-6 \pm 2\sqrt{3}}{6}$

Full Video Link



(a)  $s = -0.422$     (b)  $s = 1.573$   
 (c)  $s = 0.123$     (d)  $s = -4.62$   
 UKPSC AE 2007, Paper-I

(A)

Q.7].  $G(s) = \frac{k}{s(s+1)(s+2)}$

C.E =  $s(s+1)(s+2) + k = 0$

$s(s^2 + 3s + 2) = -k$

$s^3 + 3s^2 + 2s = -k$

$3s^2 + 6s + 2 = \frac{-dk}{ds} = 0$

$s = \frac{-6 \pm \sqrt{36 - 4(3)(2)}}{2(3)}$

$s = -1 \pm 0.577$

$s = -1 + 0.577$

$= -0.423$

$s = -1 - 0.577$

$= -1.577$

$\times s = -1.577$

$= \frac{-6 \pm \sqrt{36 - 24}}{6}$

$= \frac{-6 \pm \sqrt{12}}{6}$

$s = \frac{-6 \pm 2\sqrt{3}}{6}$

$s = \frac{-6}{6} \pm \frac{2\sqrt{3}}{6}$

$= -1 \pm \frac{1.732}{3}$

$1.000$

$0.577$

$-0.423$



$2 \overline{) 12}$   
 $2 \overline{) 6}$   
 $3$



The open loop transfer function of a system is

$$G(s)H(s) = \frac{16(s+1)}{s(s+2)(s+4)}$$

The centroid and angles of asymptotes are :

- (a)  $-2.5, 45^\circ, -45^\circ$       (b)  $-3, 60^\circ, -60^\circ$   
(c)  $-2.5, 90^\circ, 270^\circ$       (d)  $-2.5, 30^\circ, -30^\circ$

APTRANSCO AE 2019

$$= \frac{-6+1}{2} = \frac{-5}{2} = -2.5$$

Q.8]. poles  $\Rightarrow s=0; s=-2; s=-4$  } 3

Zeros  $\Rightarrow s=-1$  } 1

(C)

$$\sigma = \frac{\sum P(\text{real}) - \sum Z(\text{real})}{P-Z} = \frac{0-2-4-(-1)}{3-1} = \frac{-6+1}{2} = -2.5$$

$$\theta_A = \frac{(2q+1)180^\circ}{P-Z}$$

$$q = 0, 1, 2, \dots, P-Z-1$$
$$= 0, 1 /$$

$$\theta_{A1} = \frac{(2(0)+1)180^\circ}{2} = 90^\circ$$

$$\theta_{A2} = \frac{(2(1)+1)180^\circ}{2} = 270^\circ$$

Full Video Link



0.5 x

A unity feedback system has open loop transfer

$$\text{function } GH(s) = \frac{K}{s(s+4)(s+16)}.$$

Its root locus plot intersects the  $j\omega$  axis at \_\_\_\_.

- (a)  $\pm j16$                       (b)  $\pm j2$   
(c)  $\pm j4$                         (d)  $\pm j8$

**BHEL ET 2019**



Q.9]. C.E =  $s(s+4)(s+16) + k = 0$ .

$$= s(s^2 + 20s + 64) + k = 0.$$

$$s^3 + 20s^2 + 64s + k = 0.$$

$$20s^2 + k = 0.$$

Full Video Link



(c)  $\pm j4$       (d)  $\pm j8$   
**BHEL ET 2019**

Q.9]. C.E =  $s(s+4)(s+16) + k = 0$   
 $= s(s^2 + 20s + 64) + k = 0$   
 $s^3 + 20s^2 + 64s + k = 0$

$s^3$		1	64	$\Rightarrow$	$20s^2 + k = 0$	$20s^2 + 1280 = 0$
$s^2$		20	k	$\Rightarrow$	$1280 - k = 0$	$20s^2 = -1280$
$s^1$		$\frac{1280 - k}{20}$	$= 0$		$\therefore k = 1280$	$s^2 = \frac{-1280}{20}$
$s^0$		k				$s = \pm j8$



Q.10]. The Root Locus of the feedback control system having the characteristics eqn  $s^2 + 6ks + 2s + 5 = 0$  where  $k > 0$  enter into real axis at UPPCL AE 2019.

- A].  $s = 5$     B].  $s = 1$     C].  $s = -\sqrt{5}$     D].  $s = -1$

Sol: Breaking points.

**(CE)**

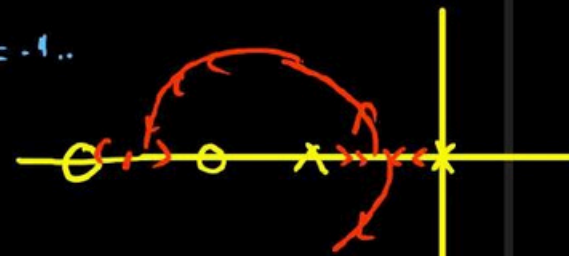
$$s^2 + 2s + 6ks + 5 = 0.$$

$$s^2 + 2s + 5 = -6ks$$

$$\frac{s^2 + 2s + 5}{6s} = -k$$

$$s = +\sqrt{5} ; -\sqrt{5}$$

$$s = -\sqrt{5}$$



$$-\frac{dk}{ds} = 0 = \frac{6s[2s+2] - [s^2+2s+5][6]}{36s^2}$$

$$12s^2 + 12s - [6s^2 + 12s + 30] = 0.$$

$$12s^2 + 12s - 6s^2 - 12s - 30 = 0.$$

$$6s^2 - 30 = 0.$$

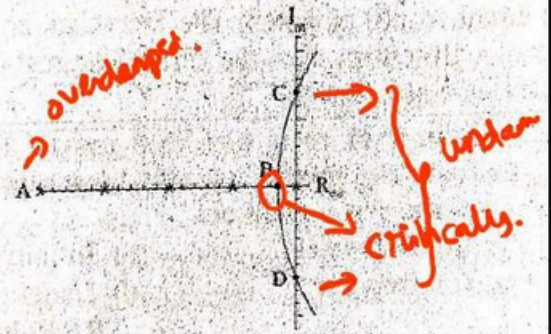
$$6s^2 = 30 \Rightarrow s^2 = \frac{30}{6} = 5$$

**(C)**



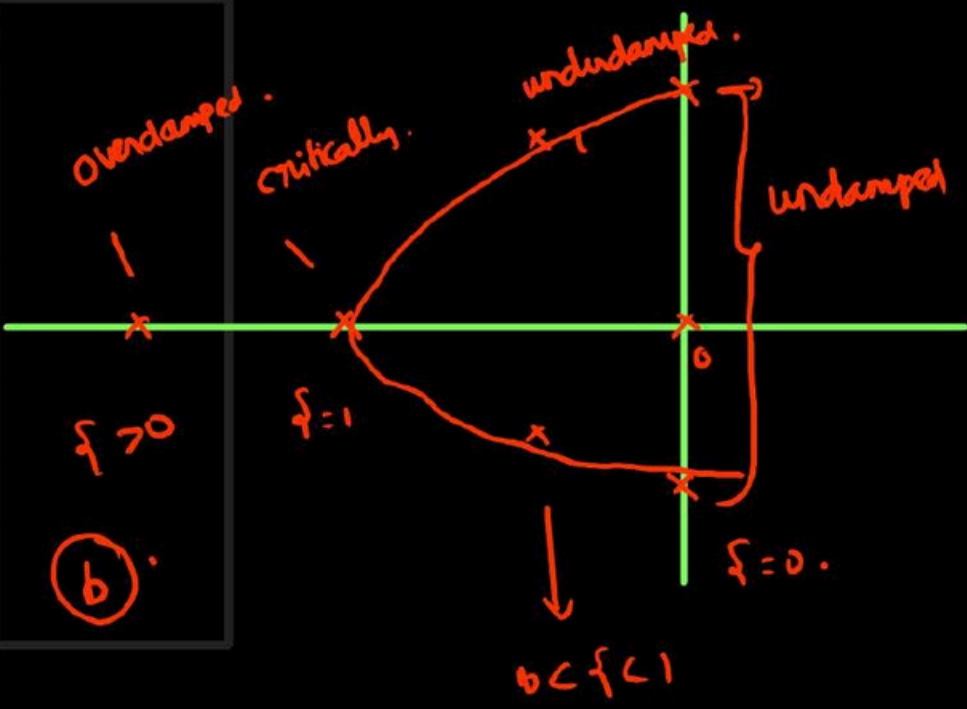


Typical root locus diagram of a system is shown below. Find the point where the system is critically damped.

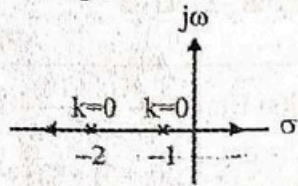


- (a) At Point 'A'
- (b) At Point 'B'
- (c) At Points 'C' and 'D'
- (d) None of the above

ISRO Scientist/Engineer 2018



The root locus of unity feedback system is shown in the figure:



The closed loop transfer function of the system

- (a)  $\frac{C(S)}{R(S)} = \frac{K}{(S+1)(S+2)}$
- (b)  $\frac{C(S)}{R(S)} = \frac{-K}{(S+1)(S+2)+K}$
- (c)  $\frac{C(S)}{R(S)} = \frac{K}{(S+1)(S+2)-K}$
- (d) None of the above

UJVNL AE 2016

$\text{Poles} \Rightarrow s = -1 \quad s = -2$

$G(s) = \frac{K}{(s+1)(s+2)}$

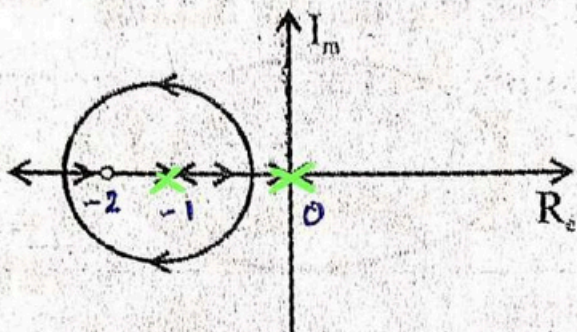
$\text{CLTF} = \frac{K}{(s+1)(s+2) + K}$

$\text{CLT} = \frac{N}{D+N} = \frac{K}{(s+1)(s+2) + K}$

~~(a)~~  
 (b)



The root locus of an unity feedback system is shown below. The open loop transfer function will be



(a)  $\frac{k(s+2)}{s(s+1)}$

(b)  $\frac{k(s+1)}{s(s+2)}$

(c)  $\frac{ks}{(s+1)(s+2)}$

(d)  $\frac{k}{s(s+1)(s+2)}$

UKPSC AE 2007, Paper-I  
ESE 1993

Q.3] poles  $\Rightarrow s = 0 \quad s = -1$

zeros  $\Rightarrow s = -2$

OLTF  $G(s) = \frac{k(s+2)}{s(s+1)}$

(A)



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If the pole-zero location is as shown in figure, then time response will be-

(a)  $\Rightarrow$  Stable.

(b)  $\Rightarrow$  Stable.

(c)  $\Rightarrow$  Stable.

(d)  $\Rightarrow$  Exponential unbounded.

Q4.

unstable.

$\sigma > 0$

(D) ن

(D)

Full Video Link



The open-loop transfer function of a feedback control system is given by-

$$\frac{G(s)}{H(s)} = \frac{K(s+2)}{s(s+4)(s^2+4s+8)}$$

One of following is a set of the centroid point coordinates, where asymptotes of the root loci of above transfer function meet in the s-plane

- (a) (-1, 0)                      (b) (-2, 0)  
 (c)  $(-\frac{10}{3}, 0)$                   (d) (2, 0)

RPSA AE 2018

Q.1]  $G(s) = \frac{K(s+2)}{s(s+4)(s^2+4s+8)}$

poles  $\Rightarrow s=0 \quad s=-4$

$s = -2 + i4 \quad s = -2 - i4$

Zeros  $\Rightarrow \boxed{s = -2}$

Asymptotes =  $P - Z$   
 $= 4 - 1 = 3$

$$\sigma = \frac{\sum P(\text{real}) - \sum Z(\text{real})}{P - Z}$$

(b)

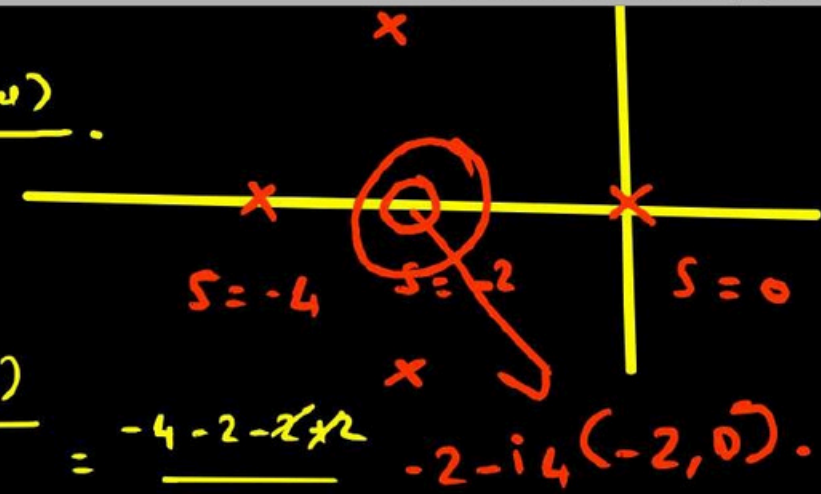
$$\sigma = \frac{0 - 4 - 2 - 2 - (-2)}{3} = \frac{-4 - 2 - 2 + 2}{3} = \frac{-6}{3} = -2$$

$$s = \frac{-4 \pm \sqrt{16 - 32}}{2}$$

$$= \frac{-4 \pm \sqrt{-16}}{2}$$

$$= \frac{-4 \pm i4}{2}$$

$$= -2 \pm i4$$



0.4 x

Full Video Link



The value of  $k$  at which the root locus crosses the imaginary axis, makes the system :-

- (a) Stable                      (b) Underdamped  
(c) Marginally stable        (d) Unstable

**ISRO Scientist/Engineer 2014**

**UKPSC AE 2013 Paper-I**

**ESE 2008**

Q.5. (c)

Full Video Link



For the following characteristic equation, the centroid of the root locus plot is  
 $s^3 + 2s^2 + ks + k = 0$   
 (a) 0.5 (b) -0.5  
 (c) -1 (d) 1  
 UKPSC AE 2013 Paper-I

(B)

Q.7.

C.E:  $s^3 + 2s^2 + \underbrace{k(s+1)} = 0$   
 D + N

OLTF:  $\frac{k}{s(s+1)(s+2)}$   
 C.E =  $s(s+1)(s+2) + k = 0$

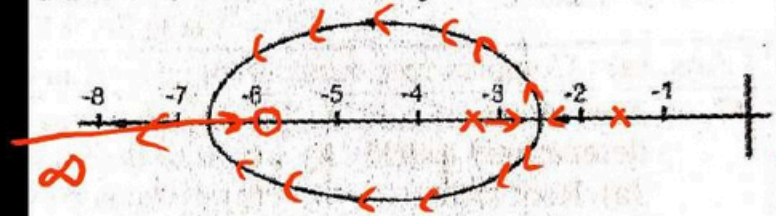
OLTF =  $\frac{k(s+1)}{s^3 + 2s^2}$

G(s) =  $\frac{k(s+1)}{s^2(s+2)}$   
 poles  $\Rightarrow s=0, 0, s=-2$   
 zero  $\Rightarrow s=-1$

$$\sigma = \frac{\sum P(\text{real}) - \sum Z(\text{real})}{p-z} = \frac{0+0-2 - (-1)}{3-1} = \frac{-2+1}{2} = \frac{-1}{2} = -0.5$$

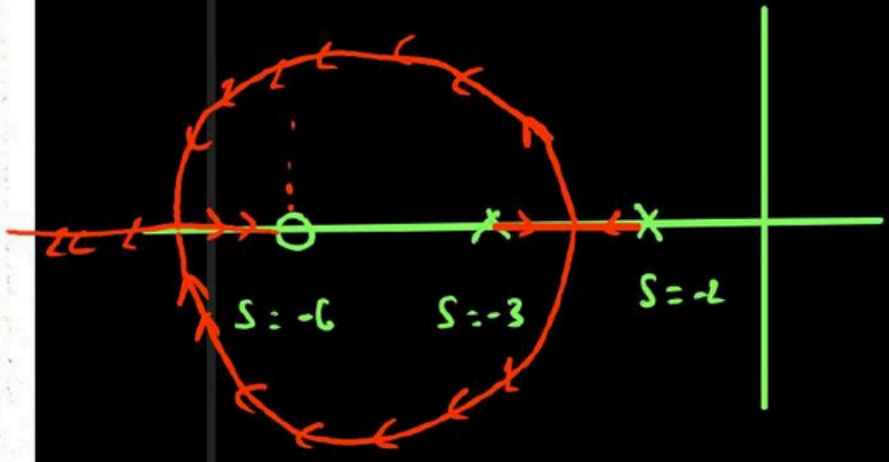


For a given root-locus, what is the open loop transfer function for unity feedback?



- (a)  $\frac{K}{S(S+2)(S+5)}$
- (b)  $\frac{K(S+3)}{(S+1)(S+5)}$
- (c)  $\frac{K(S+6)}{(S+2)(S+3)}$
- (d)  $\frac{K(S+5)}{(S+1)(S+2)}$

UPPCL AE 18-05-2016



Poles  $\Rightarrow s = -2 \quad s = -3$

Zeros  $\Rightarrow s = -6$

Q.8].  
 (c)

$$G(s) = O.L.T.F = \frac{K(S+6)}{(S+2)(S+3)}$$



Which of the following is the best method for determining the stability and transient response?

UKPSC AE 2012

- (a) Root locus
- (b) Bode plot
- (c) Nyquist plot
- (d) None of the above

Haryana PSC Civil Services (Pre) 2014

KARNATAKA AE 2016 BPSC AE 2012 Paper- VI

(A) Root Locus



The end points of root loci are <sup>x</sup>

(a) open loop poles	(b) closed loop poles $\rightarrow$
(c) open loop zeros	(d) closed loop zeros $\rightarrow$

UKPSC AE 2012, Paper-I

0.10.

(c)

Starting point  $k=0 \Rightarrow$  **OLP = CLP**  $\Rightarrow$  Open loop poles.

at  $k = \infty$  ending point **CLP = OLZ**  $\Rightarrow$  open loop zeros.





# Frequency Response.

## 1. Bode Plot.

Decade :  $\omega_{next} = 10 \omega_{previous}$ .  $\frac{\omega_2}{\omega_1} = 10$ .

Octave :  $\omega_{next} = 2 \omega_{previous}$ .  $\frac{\omega_2}{\omega_1} = 2$ .

Relationship b/w Decade and octave:

$$20 \text{ dB/dec} = 6 \text{ dB/octave.}$$

40dB/decade.

12dB/octave.

Two Plots:

\* Magnitude Plot

\* phase Plot.

### Important Plots:

$n$ -poles at origin	$n$ -zeros at origin
* Magnitude	* Magnitude



## Important Plots:

$n$ -poles at origin	$n$ -zeros at origin
* $  \text{Magnitude}  _{\text{dB}} = -20n \log w$	* $  \text{Magnitude}  _{\text{dB}} = +20n \log w$
* Slope = $-20n \text{ dB/decade}$ = $-6n \text{ dB/octave}$	* Slope = $20n \text{ dB/decade}$ = $6n \text{ dB/octave}$
* Phase = $-\frac{n\pi}{2}$	* Phase = $\frac{n\pi}{2}$
* poles always give negative slope.	* zeros always give positive slope.

\* Corner Frequency  $w = \frac{1}{\tau}$  😊

Phase Margin and Gain Margin.

Full Video Link



1.4 x

## Phase Margin and Gain Margin.

$$\text{Gain Crossover Frequency } \omega_{GCF} \quad |G_H(j\omega)|_{\omega=\omega_{GCF}} = 1.$$

$$20 \log |G_H(j\omega)|_{\omega=\omega_{GCF}} = 0 \text{ dB}.$$

$$\text{Phase Crossover Frequency } \omega_{PCF} \quad \angle G_H(j\omega)|_{\omega=\omega_{PCF}} = -180^\circ.$$

$$\text{Gain Margin} = -20 \log (|G_H(j\omega)|_{\omega=\omega_{PCF}}).$$

$$\text{Phase Margin} = 180 + \angle G_H(j\omega)|_{\omega=\omega_{GCF}}.$$

### Important Points:

- \* Minimum Phase System [finite poles and zeroes lies in LH of s-plane].
- \* Non-Minimum Phase System [one (or) more zeroes/poles lies in RH of s-plane].
- \* All pass system [all zeroes lies in Right and all poles lies in Left half of s-plane].

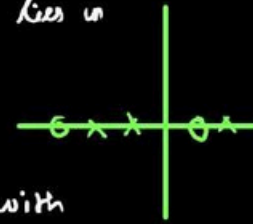


\* Minimum Phase System + Non-Minimum phase System with

Full Video Link



\* All pass System [ all zeroes lies in Right and all poles lies in Left half of s-plane ].



\* Minimum phase System + Non-Minimum phase System with Zero in RH of s-plane.

- \*  $GM > 0\text{ dB}$  ;  $PM > 0\text{ dB}$   $\Rightarrow$  Stable.  $\Rightarrow GM$  and  $PM > 0$
- \*  $GM = 0\text{ dB}$  ;  $PM = 0\text{ dB}$   $\Rightarrow$  Marginally.  $\Rightarrow GM$  and  $PM = 0$ .
- \*  $GM < 0\text{ dB}$  ;  $PM < 0\text{ dB}$   $\Rightarrow$  unstable.  $\Rightarrow GM$  and  $PM < 0$ .

\* Non-Minimum phase System with poles in RH of s-plane. (2/1)

- \*  $GM < 0\text{ dB}$  ;  $PM < 0\text{ dB}$   $\Rightarrow$  Stable.  $\Rightarrow GM, PM < 0 \Rightarrow$  Stable.
- \*  $GM = 0\text{ dB}$  ;  $PM = 0\text{ dB}$   $\Rightarrow$  Marginally.
- \*  $GM > 0\text{ dB}$  ;  $PM > 0\text{ dB}$   $\Rightarrow$  unstable.  $\Rightarrow GM, PM > 0 \Rightarrow$  unstable.

\*  $\tan^{-1} A + \tan^{-1} B = \tan^{-1} \frac{A+B}{1-AB}$

Handwritten notes and formulas in a toolbar area:  $\frac{A+B}{1-AB}$ ,  $+ AB > 1 \Rightarrow$  then add with  $\frac{A-B}{1+AB}$ .



\*  $G_M < 0 \text{ dB}$  ;  $P_M < 0 \text{ dB}$   $\Rightarrow$  unstable.  $\Rightarrow$

\* Non-Minimum Phase System with poles in RH of s-plane. (2%)

\*  $G_M < 0 \text{ dB}$  ;  $P_M < 0 \text{ dB}$   $\Rightarrow$  stable.  $\Rightarrow G_M, P_M < 0 \Rightarrow$  stable.

\*  $G_M = 0 \text{ dB}$  ;  $P_M = 0 \text{ dB}$   $\Rightarrow$  Marginally.

\*  $G_M > 0 \text{ dB}$  ;  $P_M > 0 \text{ dB}$   $\Rightarrow$  unstable.  $\Rightarrow G_M, P_M > 0 \Rightarrow$  unstable.

$$* \tan^{-1} A + \tan^{-1} B = \tan^{-1} \left[ \frac{A+B}{1-AB} \right] \quad AB > 1 \rightarrow \text{then add with } \pi.$$

$$* \tan^{-1} A - \tan^{-1} B = \tan^{-1} \left[ \frac{A-B}{1+AB} \right].$$

$$* \tan^{-1} A + \tan^{-1} B + \tan^{-1} C = \tan^{-1} \left[ \frac{A+B+C - ABC}{1-AB-BC-CA} \right].$$

Full Video Link



1.4 x

## 2. Polar Plot.

\* Step 1  $\Rightarrow |M|_{\omega=0} ; \angle \phi|_{\omega=0}$ .

ii  $\Rightarrow |M|_{\omega=\infty} ; \angle \phi|_{\omega=\infty}$ .

iii  $\Rightarrow$  Starting Direction.

$\rightarrow$  Clockwise  $\Rightarrow$  Finite poles nearer to imaginary axis.

$\rightarrow$  Anticlockwise  $\Rightarrow$  Finite zero nearer to imaginary axis.

iv  $\Rightarrow$  Ending Direction.

$\rightarrow$  Clockwise  $\Rightarrow \phi_0 - \phi_{\infty} = +ve$ .

$\rightarrow$  Anticlockwise  $\Rightarrow \phi_0 - \phi_{\infty} = -ve$ .



Full Video Link



2.0 x

### 3. Nyquist Plot.

⇒ Find the close loop poles in RH of s-plane.

Important Result:

$$* 1+s = 1+j\omega = \tan^{-1}\omega.$$

$$* 1-s = 1-j\omega = \tan^{-1}\left(\frac{-\omega}{1}\right) = -\tan^{-1}\omega.$$

$$* s-1 = j\omega-1 = 180^\circ - \tan^{-1}\omega.$$

$$* -s-1 = 180^\circ + \tan^{-1}\omega.$$

\* Plot starting from ending point of polar plot.

i].



Full Video Link



\*  $s-1 = j\omega-1 = 180^\circ - \tan^{-1} \omega$ .

\*  $-s-1 = 180^\circ + \tan^{-1} \omega$ .

\* plot starting from ending point of polar plot.

i].  $s^1 \Rightarrow \pi$  encirclement.

ii].  $s^2 \Rightarrow 2\pi$  encirclement.

⋮

$s^n \Rightarrow n\pi$  encirclement.



\* For stable condition  $\rightarrow$  no poles should lie in RH of S-plane.

$N = P_+ - Z_+$  .  $Z_+ = 0$  ☺ .

$Z_+ \neq 0$ .

09:30

09:40

09:00

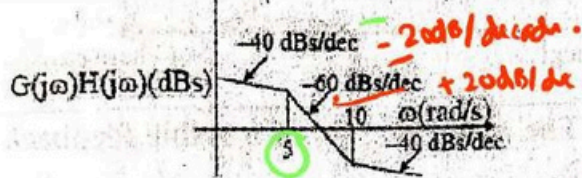
30

$P_+ \Rightarrow$  poles of OLTF.

$Z_+ \Rightarrow$  poles of CLTF.

Ob:





The open loop transfer function  $G(s)H(s)$  of a Bode's plot for feedback system as shown in figure is

(a)  $\frac{K(s+5)}{s^2(s+10)}$

(b)  $\frac{K(s+5)}{s(s+10)}$

(c)  $\frac{K(s+10)}{s^2(s+5)}$

(d)  $\frac{K(s+10)}{s(s+5)}$

ESE 2018

Q.1]  $-40 \text{ dB/dec} \Rightarrow 2 \text{ poles}$

$\omega_{c1} = 5$

$\omega_{c2} = 10$

$$T.F = \frac{K \left(1 + \frac{s}{10}\right)}{s^2 \left(1 + \frac{s}{5}\right)}$$

Ⓞ ✓

$$T.F = \frac{K(10+s)}{10 s^2 \left(\frac{5+s}{5}\right)}$$

$$T.F = \frac{K(10+s)}{10^2 s^2 (s+5)} \times \frac{5}{5}$$

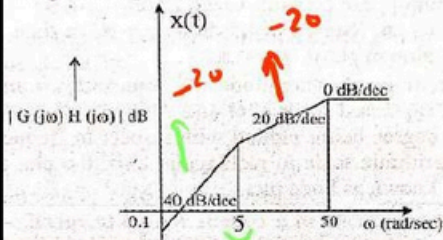
$$= \left(\frac{K}{2}\right) \cdot \frac{(10+s)}{s^2 (s+5)} = \frac{K(s+10)}{s^2 (s+5)}$$

Full Video Link



0.5 x

The open-loop transfer function for the Bode's magnitude plot is



- (a)  $G(s)H(s) = \frac{K}{s^2(1+0.2s)(1+0.02s)}$
  - (b)  $G(s)H(s) = \frac{Ks}{(1+0.2s)(1+0.02s)}$
  - (c)  $G(s)H(s) = \frac{Ks^2}{(s+5)(s+50)}$
  - (d)  $G(s)H(s) = \frac{K}{s^2(s+5)(s+50)}$
- ESE 2017

(c)

Q.2] Initial slope = +40dB/decade.

$\omega_{c1} = 5 \quad \omega_{c2} = 50.$

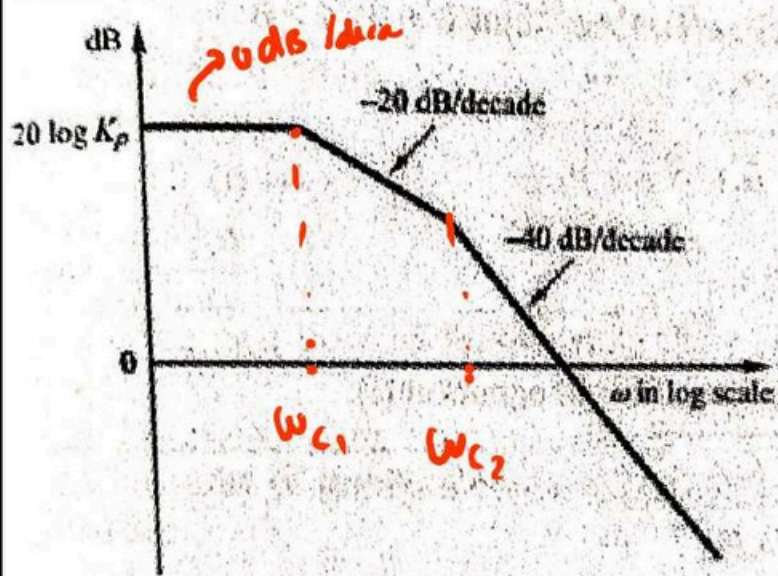
$$T.F = \frac{Ks^2}{\left(1 + \frac{s}{5}\right) \left(1 + \frac{s}{50}\right)}$$

$$= \frac{Ks^2}{\left(\frac{5+s}{5}\right) \left(\frac{50+s}{50}\right)} = \frac{250Ks^2}{(s+5)(s+50)}$$

$$= \frac{Ks^2}{(s+5)(s+50)}$$



If a system has the following ABM plot, what is the order of the system?



- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 5

CGPSC AE 2017

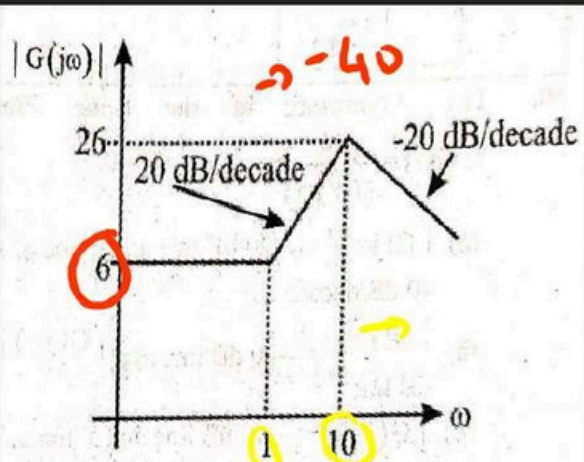
Q.3]. Initial 0 dB

$$T.F : O.T.F = \frac{k}{\dots}$$

$$\left(1 + \frac{s}{\omega_{c1}}\right) \left(1 + \frac{s}{\omega_{c2}}\right)$$

Order = no. of pole  
= 2.





The transfer function of the above bode plot may be :

- (a)  $\frac{2(1+s)^2}{\left(1+\frac{s}{10}\right)^2}$  ~~X~~
- (b)  $\frac{2(1+s)}{\left(1+\frac{s}{10}\right)^2}$  ✓
- (c)  $\frac{2(1+s)}{\left(1+\frac{s}{10}\right)}$  ~~X~~
- (d)  $\frac{2(1+s)}{(1+10s)^2}$  ~~X~~

OPSC AEE 2015 Paper-I

Q.4]. T.F =  $\frac{2}{k} (1+s)$

$\left(1+\frac{s}{10}\right)^2$

B

$2 = \log k$

$(10)^2 = k \Rightarrow k = 100$

$3 = \log k$

$\frac{6^3}{20} = \log k \Rightarrow (10)^{\frac{3}{10}} = k \cdot (10)^3 = k$

$4 = \log k$

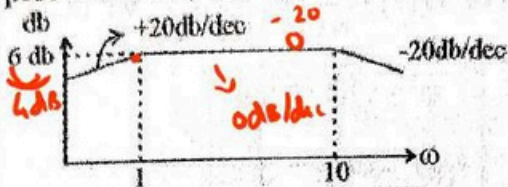
$\therefore k = 2.99$

1, ... n

$\therefore k = 2$



The transfer function of the system whose Bode plot is shown, will be



- (a)  $10 s / (s+1)(s+10)$  ✓
- (b)  $20 s / (s+1)(s+10)$  ✓
- (c)  $10 / (s+1)^2 (s+10)$  ✗
- (d)  $20 / (s+1)(s+10)^2$  ✗

UKPSC AE 2013 Paper-I

**B**

Q.5]. T.F =  $\frac{k s}{(1+s)(1+\frac{s}{10})} = \frac{20s}{(1+s)(10+s)}$

$6dB \Rightarrow \omega = 1$

$6 = 20 \log [ks]$

$= 20 \log k + 20 \log [\omega]$

$= 20 \log k + 20 \log [1]$

$\frac{6}{20} = \log k$

$k = 2$

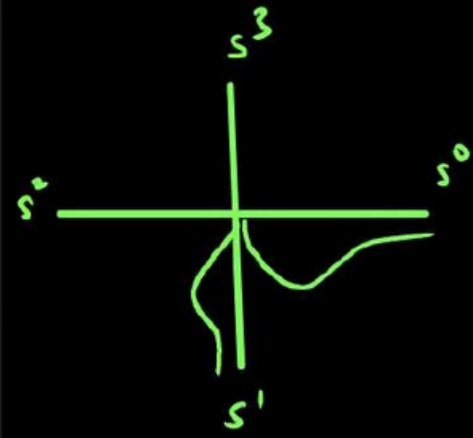


If poles are added at the origin, then the polar plot gets shifted by \_\_\_\_\_ at  $\omega = 0$ .

(a)  $-90^\circ$  (b)  $-180^\circ$   
(c)  $-60^\circ$  (d)  $-45^\circ$

APPSC Poly. Tech. Lect. 2020

Q.17.  $-90^\circ$ . (A)



Calculate the gain margin of the system  $G(s)$  with unity feedback. Where  $G(s)$  is given as

$$G(s) = \frac{5}{s(s+5)(s+15)}$$

- (a)  $20 \log_{10} 500$  dB      (b)  $20 \log_{10} 300$  dB  
(c)  $20 \log_{10} 50$  dB      (d)  $20 \log_{10} 100$  dB

DMRC AM 2020

(B)

Q.2]  $G(s) = \frac{5k}{s(s+5)(s+15)}$

$$GM = 20 \log_{10}(300)$$

$$C.F \Rightarrow s(s^2 + 20s + 75) + 5k = 0$$

$$s^3 + 20s^2 + 75s + 5k = 0$$

$$k = 300$$

$s^3$		1	75
$s^2$		20	$5k$
$s^1$		$\frac{1500 - 5k}{20}$	
$s^0$		$5k$	

$$\frac{1500 - 5k}{20} = 0$$

$$1500 = 5k$$

$$\therefore k = \frac{300}{1}$$

Full Video Link



0.3 x

Obtain the corner frequency of the transfer function are :

$$G(s) = \frac{20(0.1s+1)}{s^2(0.2s+1)(0.02s+1)} \rightarrow$$

- (a) 15 rad/sec, 25 rad/sec, 50 rad/sec
- (b) 100 rad/sec, 50 rad/sec, 5000 rad/sec
- (c) 1 rad/sec, 15 rad/sec, 150 sec
- (d) 10 rad/sec, 5 rad/sec, 50 rad/sec

CIL MT 2020

Q.3].  $G(s) = \frac{20 \left( 1 + \frac{s}{\omega_{c1}} \right)}{s^2 \left( 1 + \frac{s}{\omega_{c2}} \right) \left( 1 + \frac{s}{\omega_{c3}} \right)}$

$$\rightarrow \omega_{c1} = \frac{50}{100} = 50.$$

$\therefore \omega_{c1} = 10, 5, 50$

$$\frac{s}{\omega_{c1}} = 0.1s$$

(D)

$$\omega_{c1} = \frac{1}{0.1} = 10.$$

$$\frac{s}{\omega_{c2}} = 0.2s \Rightarrow \omega_{c2} = \frac{1}{0.2} = \frac{10}{2} = 5.$$

$$\frac{s}{\omega_{c3}} = 0.02s \Rightarrow \omega_{c3} = \frac{1}{0.02}$$

$$\omega_{c3} = \frac{50}{100} = 50.$$

Full Video Link



0.4 x

For the given transfer function:

$$G(s) = \frac{50}{s(1+0.25s)(1+0.1s)}$$

The corner frequencies will be respectively:

- (a) 2 rad/sec and 10 rad/sec
- (b) 4 rad/sec and 10 rad/sec
- (c) 2 rad/sec and 5 rad/sec
- (d) 4 rad/sec and 5 rad/sec

UPSC Poly. Lect. 2019

Q.4. Corner Frequency.

(b)

$$0.1s = \frac{s}{\omega_c} \Rightarrow \omega_c = \frac{1}{0.1} = \frac{100}{25} = 4 \text{ rad/sec.}$$

= 10 rad/sec.

$$1+0.25s \Rightarrow 1 + \frac{s}{\omega_{c1}}$$

$$0.25s = \frac{s}{\omega_{c1}}$$

$$\omega_{c1} = \frac{1}{0.25}$$

Full Video Link



0.4 x

The transfer function  $G(S) = \frac{1}{s^2}$  has a 0 dB crossing in its Bode magnitude plot at  
 (a) 100 rad/s (b) 10 rad/s  
 (c) 0.1 rad/s (d) 1 rad/s  
 UPPCL AE 01-01-2019 Shift I

$K=1$   $K=2$   $20 \log 2$   
 $20 \log 1 = 0 \text{ dB}$

Q.5]  $G(s) = \frac{1}{(j\omega)^2}$

$|G(j\omega)| = 1$

$20 \log |G(j\omega)| = 0 \text{ dB}$

$\left| \frac{1}{\omega^2} \right| = 1 \Rightarrow \frac{1}{\omega^2} = 1$

$G(s) = \frac{1}{s(s+1)}$

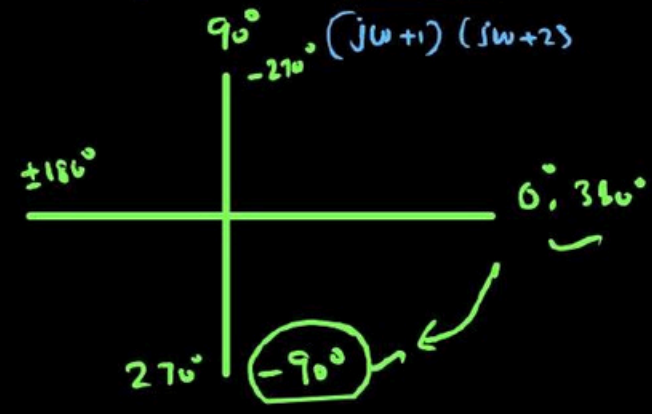
$\omega^2 = 1$   
 $\omega = 1 \text{ rad/sec}$



The minimum phase attained for the frequency response of a causal system  $G(s) = \frac{s+10}{(s+1)(s+2)}$  as the frequency varies from 0 to  $\infty$  rad/s is  
 (a) 180 degrees (b) 90 degrees  
 (c) -90 degrees (d) -180 degrees  
 UPPCL AE 01-01-2019 Shift I

Q.6.  $G(s) = \frac{s+10}{(s+1)(s+2)}$

$\angle G(j\omega) = \frac{j\omega+10}{(j\omega+1)(j\omega+2)}$



$\angle G(j\omega) = \tan^{-1} \frac{\omega}{10} - \tan^{-1} \omega - \tan^{-1} \frac{\omega}{2}$  (C)

$\omega = 0^\circ:$

$\angle G(j\omega) = \tan^{-1} 0 - \tan^{-1} 0 - \tan^{-1} 0 = 0^\circ$   
 $0^\circ, -90^\circ$

$\omega = \infty:$

$\angle G(j\omega) = \tan^{-1} \infty - \tan^{-1} \infty - \tan^{-1} \infty = -90^\circ$   
 $0^\circ, -90^\circ$





Which of the following transfer function has the given polar plot?

(a)  $G(s) = \frac{(s+\omega_1)(s+\omega_2)}{s^2(s+\omega_2)(s+\omega_3)}$  - X

(b)  $G(s) = \frac{s+\omega_1}{s(s+\omega_2)(s+\omega_3)}$  ✓

(c)  $G(s) = \frac{s+\omega_1}{s(s+\omega_2)}$

(d)  $G(s) = \frac{1}{s(s+\omega_2)(s+\omega_3)}$

UPPCL AE 04-11-2019 Shift II

(b).  $G(j\omega) = \frac{j\omega + \omega_1}{j\omega(j\omega + \omega_2)(j\omega + \omega_3)}$       $\sqrt{\omega^2} = \omega$

$|G(j\omega)| = \frac{\sqrt{\omega^2 + \omega_1^2}}{\omega \sqrt{\omega^2 + \omega_2^2} \sqrt{\omega^2 + \omega_3^2}}$

At  $\omega=0 \Rightarrow |G(j\omega)| = \frac{\sqrt{\omega_1^2}}{\omega \sqrt{\omega_2^2} \sqrt{\omega_3^2}} = \infty$

At  $\omega \rightarrow \infty$

$|M|_{\omega=0} \geq |M|_{\omega=\infty}$

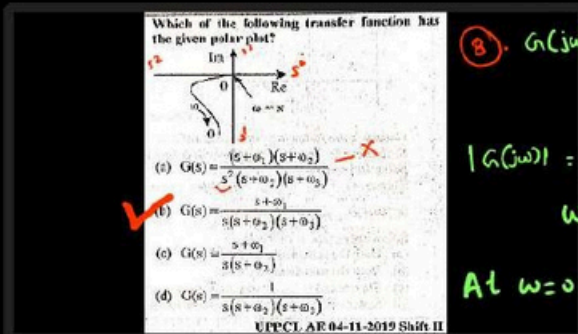
$|G(j\omega)| = \frac{\sqrt{\omega^2 + \omega_1^2}}{\omega \sqrt{\omega^2 + \omega_2^2} \sqrt{\omega^2 + \omega_3^2}} = 0$

$\angle G(j\omega) = \tan^{-1} \frac{\omega}{\omega_1} - \tan^{-1} \omega - \tan^{-1} \frac{\omega}{\omega_2} - \tan^{-1} \frac{\omega}{\omega_3}$

$\omega=0 \Rightarrow \angle G(j\omega) = 0 - 90^\circ - 0 - 0 = -90^\circ$

$\omega = \infty \Rightarrow \angle G(j\omega) = \tan^{-1} \frac{\omega}{\omega_1} - \tan^{-1} \omega - \tan^{-1} \omega - \tan^{-1} \omega$





8.  $G(j\omega) = \frac{j\omega + \omega_1}{j\omega(j\omega + \omega_2)(j\omega + \omega_3)}$       $\sqrt{\omega^2} = \omega$

$$|G(j\omega)| = \frac{\sqrt{\omega^2 + \omega_1^2}}{\omega \sqrt{\omega^2 + \omega_2^2} \sqrt{\omega^2 + \omega_3^2}}$$

At  $\omega = 0 \Rightarrow |G(j\omega)| = \frac{\sqrt{\omega_1^2}}{\omega \sqrt{\omega_2^2} \sqrt{\omega_3^2}} = \infty$

$\tan^{-1} \frac{\omega}{\omega_2}$

$0 - 90^\circ - \tan^{-1} \omega = -90^\circ$

$\tan^{-1} \frac{\omega}{\omega_2} - \tan^{-1} \omega - \tan^{-1} \omega$

$-90^\circ$

At  $\omega = \infty$

$$|G(j\omega)| = \frac{\sqrt{\omega + \omega_1^2}}{\omega \sqrt{\omega + \omega_2^2} \sqrt{\omega + \omega_3^2}} = 0$$

$|M|_{\omega=0} \geq |M|_{\omega=\infty}$

$$\angle G(j\omega) = \tan^{-1} \frac{\omega}{\omega_1} - \tan^{-1} \omega - \tan^{-1} \frac{\omega}{\omega_2} - \tan^{-1} \frac{\omega}{\omega_3}$$

$\omega = 0 \Rightarrow \angle G(j\omega) = 0 - 90^\circ - 0 - 0 = -90^\circ$

$\omega = \infty \Rightarrow \angle G(j\omega) = \tan^{-1} \frac{\omega}{\omega_1} - \tan^{-1} \omega - \tan^{-1} \omega - \tan^{-1} \omega$

$$= -90^\circ - 90^\circ = -180^\circ$$

EP:  $\phi_\infty - \phi_0$

$$= -90^\circ + 180^\circ = +90^\circ$$



$$C. G(j\omega) = \frac{j\omega + \omega_1}{j\omega(j\omega + \omega_2)}$$

$$|M| = \frac{\sqrt{\omega^2 + \omega_1^2}}{\omega \sqrt{\omega^2 + \omega_2^2}}$$

$$\boxed{\omega=0} \Rightarrow |M| = \infty$$

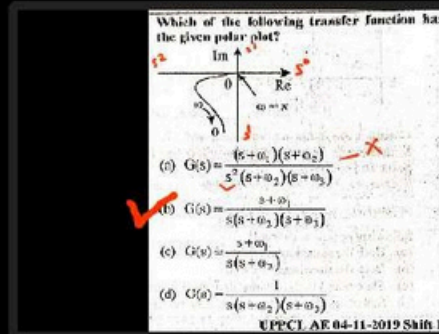
$$\boxed{\omega=\infty} \Rightarrow |M| = 0$$

$$\angle G_H(j\omega) = \tan^{-1} \frac{\omega}{\omega_1} - \tan^{-1} \omega - \tan^{-1} \frac{\omega}{\omega_2}$$

$$\boxed{\omega=0} \Rightarrow \angle G_H(j\omega) = \tan^{-1} 0 - 90^\circ - \tan^{-1} 0 = -90^\circ$$

$$\boxed{\omega=\infty} \Rightarrow \angle G_H(j\omega) = \tan^{-1} \infty - \tan^{-1} \infty - \tan^{-1} \infty = -90^\circ$$

$$FD \Rightarrow \Phi_0 - \Phi_\infty = -90^\circ - (-90^\circ) = 0$$



$$B. G(j\omega) = \frac{j\omega + \omega_1}{j\omega(j\omega + \omega_2)(j\omega + \omega_3)} \quad \sqrt{\omega^2} = \omega$$

$$|G(j\omega)| = \frac{\sqrt{\omega^2 + \omega_1^2}}{\omega \sqrt{\omega^2 + \omega_2^2} \sqrt{\omega^2 + \omega_3^2}}$$

$$At \omega=0 \Rightarrow |G(j\omega)| = \frac{\sqrt{\omega_1^2}}{\omega} = \infty$$

$$At \omega=\infty$$

$$|G(j\omega)| = \frac{\sqrt{\omega_1^2} \sqrt{\omega_2^2} \sqrt{\omega_3^2}}{\omega \sqrt{\omega + \omega_1^2} \sqrt{\omega + \omega_2^2} \sqrt{\omega + \omega_3^2}} = 0$$

$$|M|_{\omega=0} \geq |M|_{\omega=\infty}$$

$$\angle G(j\omega) = \tan^{-1} \frac{\omega}{\omega_1} - \tan^{-1} \omega - \tan^{-1} \frac{\omega}{\omega_2} - \tan^{-1} \frac{\omega}{\omega_3}$$

$$\omega=0 \Rightarrow \angle G(j\omega) = 0 - 90^\circ - 0 - 0 = -90^\circ$$

$$\omega=\infty \Rightarrow \angle G(j\omega) = \tan^{-1} \infty - \tan^{-1} \infty - \tan^{-1} \infty - \tan^{-1} \infty = -90^\circ - 90^\circ = -180^\circ$$

$$EP: \Phi_0 - \Phi_\infty = -90^\circ + 180^\circ = +90^\circ$$

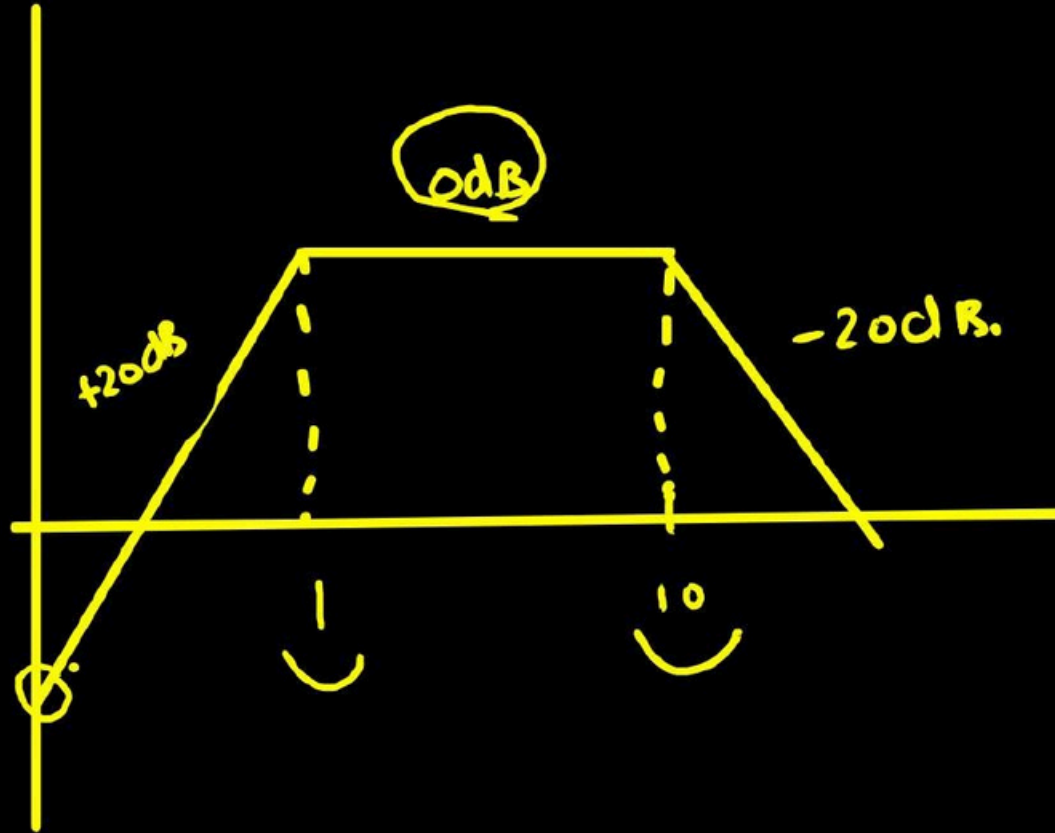


The slope of Bode gain plot of transfer function

$\frac{Ks}{(s+1)(s+10)}$  between  $\omega = 1$  and  $\omega = 10$  is:

- (a) +20 dB/decade
- (b) -40 dB/decade
- (c) zero
- (d) -20 dB/decade

BHEL ET 2019



Q.9.

$(s+10)$   
 $10 \left(1 + \frac{s}{10}\right)$

(c)



The stability criteria for the minimum phase system is:

- (a) phase margin should be positive and gain margin negative.
- (b) both gain margin and phase margin should be positive
- (c) both gain margin and phase margin should be negative
- (d) phase margin should be negative and gain margin positive.

$PM, GM > 0$

B

DSSSB AE 2019  
ESE 2007



**The Bode plot is expressed as:**

- (a)  $-6$  db/octave
- (b)  $-7$  db/octave
- (c)  $-8$  db/octave
- (d)  $-6$  db/decade

**APPSC Poly. Tech. Lect. 2020**

Q.11]. **A**



Consider the system

$$G(s)H(s) = \frac{1}{s(1+3s)(1+5s)}$$

Select the appropriate phase crossover frequency from the following options.

(a) 6 rad/sec                      (b) 2.46 rad/se  
 (c) 3.23 rad/sec                  (d) 0.2582 rad/sec

DMRC AM 2020

$$15s^3 + s = 0$$

$$s [15s^2 + 1] = 0 \quad \omega = \pm \sqrt{\frac{1}{15}}$$

$$15s^2 = -1 \quad \omega = \pm 0.2582$$

$$s^2 = \frac{-1}{15} \quad (d)$$

$$C.E \Rightarrow s(1+5s+3s+15s^2)+1=0$$

$$s + 8s^2 + 15s^3 + 1 = 0$$

$$15s^3 + 8s^2 + s + 1 = 0$$

$$s = \pm i \sqrt{\frac{1}{15}}$$

$$s = j\omega = \pm j \sqrt{\frac{1}{15}}$$



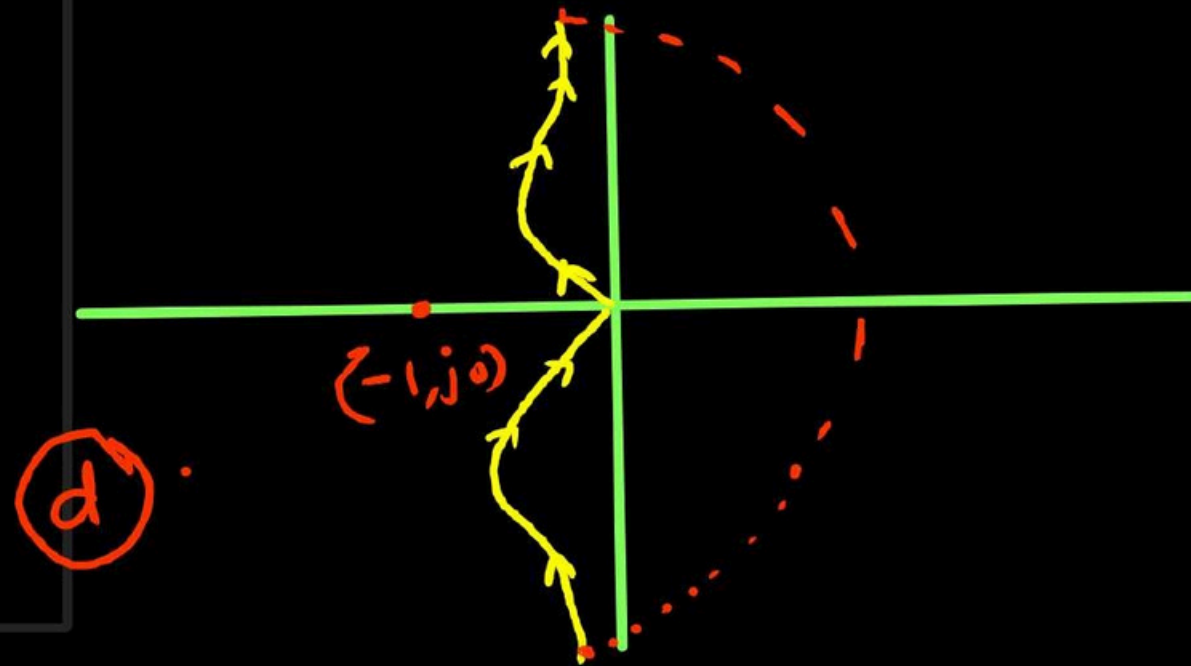


The Nyquist plot for the closed-loop control system with the loop transfer function  $G(s)H(s) = \frac{100}{s(s+10)}$  is plotted, then the critical point  $(-1, j0)$  is:

- (a) Enclosed under certain conditions
- (b) Always enclosed
- (c) Just touched
- (d) Never enclosed

UPPCL AE 18-05-2016  
ESE 2004

$$GH(j\omega) = \frac{100}{j\omega(j\omega+10)}$$



Full Video Link



0.5 x

$$G(s) = \frac{s}{1+s}$$

(B)

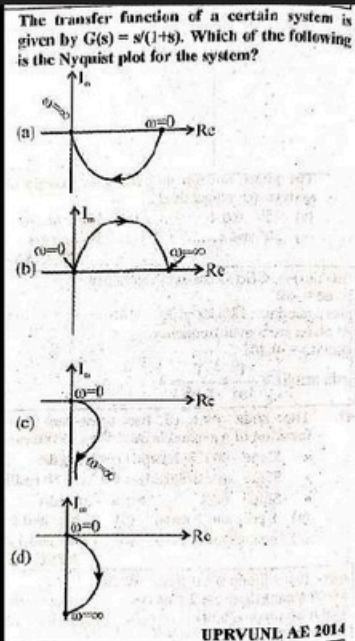
$$|M|_{\omega=0} = 0$$

$$|M|_{\omega=\infty} = 1$$

$$\angle G_H(j\omega) = 90^\circ - \tan^{-1} \omega$$

$$\angle G_H(j\omega) |_{\omega=0} = 90^\circ$$

$$\angle G_H(j\omega) |_{\omega=\infty} = 90^\circ - \tan^{-1} \infty = 90^\circ - 90^\circ = 0^\circ$$



Q.3].  $G(j\omega) = \frac{j\omega}{1+j\omega}$

$$|G(j\omega)| = \frac{\omega}{\sqrt{1+\omega^2}}$$

$$|M|_{\omega=0} = 0$$

$$|M|_{\omega=\infty} = \frac{\infty}{\infty} \text{ [indeterminate]}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1^2 + \frac{1}{\omega^2}}}$$

$$|M|_{\omega=\infty} = \frac{1}{\sqrt{1 + \frac{1}{\infty}}} = \frac{1}{\sqrt{1+0}} = 1$$

$$G(s) = \frac{s}{s(1 + \frac{1}{s})}$$

$$G(j\omega) = \frac{j\omega}{j\omega(1 + \frac{1}{j\omega})}$$

$$G(j\omega) = \frac{1}{1 + \frac{1}{j\omega}}$$



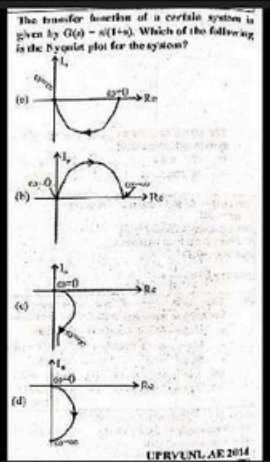
0.5 x

Full Video Link



$$G(s) = \frac{s}{1+s}$$

(B)



$$|M|_{\omega=0} = 0$$

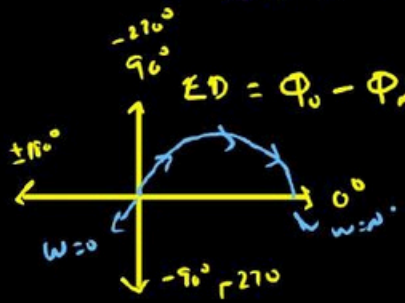
$$|M|_{\omega=\infty} = 1$$

$$\angle G_H(j\omega) = 90^\circ - \tan^{-1}\omega$$

$$\angle G_H(j\omega)|_{\omega=0} = 90^\circ$$

$$\angle G_H(j\omega)|_{\omega=\infty} = 90^\circ - \tan^{-1}\infty = 90^\circ - 90^\circ = 0^\circ$$

$$ED = \Phi_0 - \Phi_\infty = 90^\circ - 0^\circ = 90^\circ$$



Q.3]  $G(j\omega) = \frac{j\omega}{1+j\omega}$

$$|G(j\omega)| = \frac{\omega}{\sqrt{1+\omega^2}}$$

$$|M|_{\omega=0} = 0$$

$$|M|_{\omega=\infty} = \frac{\infty}{\infty} \text{ [indeterminate]}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1^2 + \frac{1}{\omega^2}}}$$

$$|M|_{\omega=\infty} = \frac{1}{\sqrt{1 + \frac{1}{\infty}}} = \frac{1}{\sqrt{1+0}} = 1$$

$$G(s) = \frac{s}{s(1 + \frac{1}{s})}$$

$$G(j\omega) = \frac{j\omega}{j\omega(1 + \frac{1}{j\omega})}$$

$$G(j\omega) = \frac{1}{1 + \frac{1}{j\omega}}$$



Consider the Nyquist plot of a servo system shown in the following figure. What would be the root loci for the system?

(a)

(b)

(c)

(d) Root loci diagram of the system cannot be drawn

UPRVUNL AE 2014

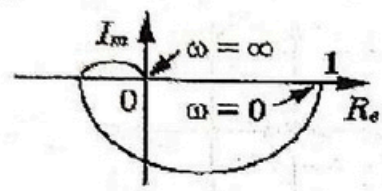
Q.4]  $G_H(s) = \frac{k}{s(s+1)}$

(B)

$s = -1$        $s = 0$



The transfer function of the following polar plot is



(a)  $G(s) = \frac{1}{(1+sT_1)(1+sT_2)}$

(b)  $G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$

(c)  $G(s) = \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$

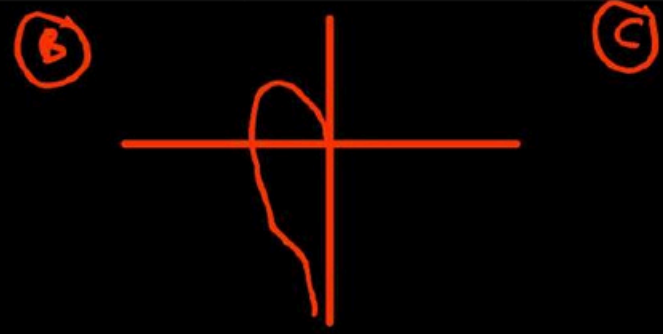
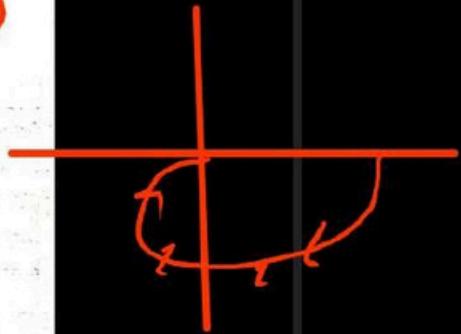
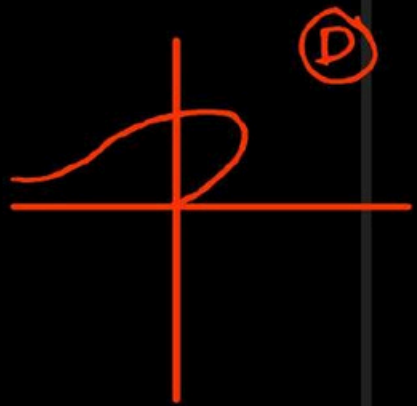
(d)  $G(s) = \frac{1}{s^2(1+sT_1)(1+sT_2)}$

TANGEDCO AE 2018

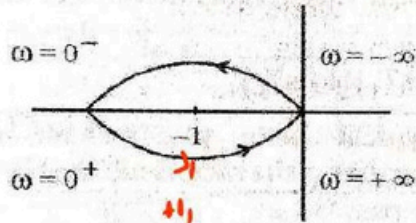
0.5].

$$G(s) = \frac{k}{(1+sT_1)(1+sT_2)(1+sT_3)}$$

(C)

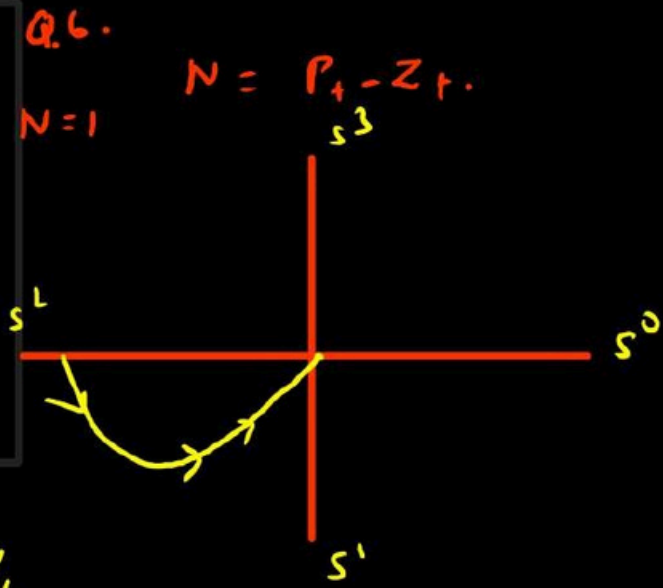


The feedback system whose Nyquist plot is shown below, is

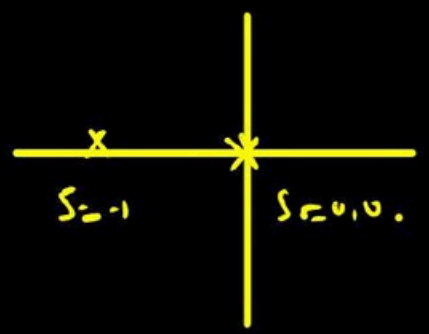


- (a) unstable
- (b) conditionally stable
- (c) stable
- (d) none of these

UKPSC AE 2012, Paper-I



T.F =  $\frac{1}{s^2(s+1)}$  //



A system has 12 poles and 2 zeros. Its high frequency asymptote in its magnitude plot will have a slope of

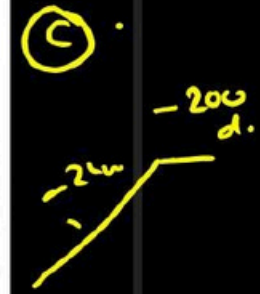
(a)  $-60$  db/dec

(b)  $-120$  db/dec

(c)  $-200$  db/dec

(d)  $-240$  db/dec

APSPDCL AE 2012



Q.7]. 12 poles  $\Rightarrow$  Slope  $\Rightarrow 12 \times (-20 \text{ dB/dec}) = -240 \text{ dB/dec}$

2 Zeros  $\Rightarrow$  Slope  $\Rightarrow 2 \times (+20 \text{ dB/dec}) = 40 \text{ dB/dec}$

Full Video Link



0.7 x



Due to adding a pole at  $s = 0$ , the angle of shifting in Nyquist plot of the system will:

- (a) shift  $90^\circ$  clockwise
- (b) shift  $180^\circ$  anticlockwise
- (c) shift  $90^\circ$  anticlockwise
- (d)  $0^\circ$  shift

A

APPSC Poly. Tech. Lect. 2020

Q.9

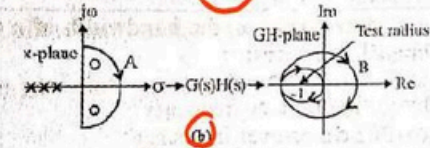
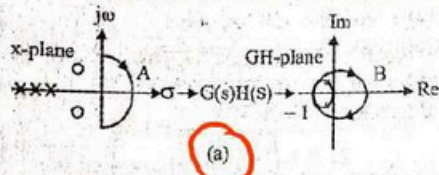
90° clockwise ⇒

Full Video Link



0.7 x

Which pair of statements applies for figure (a) and figure (b)?



- (a) Figure (a) has 2 closed loop poles and is stable.
- X Figure (b) has 0 closed loop poles and is unstable.
- (b) Figure (a) has 0 closed loop poles and is unstable.
- X Figure (b) has 2 closed loop poles and is stable.
- (c) Figure (a) has 2 closed loop poles and is unstable.
- X Figure (b) has 0 closed loop poles and is stable.
- (d) Figure (a) has 0 closed loop poles and is stable.
- Figure (b) has 2 closed loop poles and is unstable.

OMC Deputy manager 2019

Q.10].

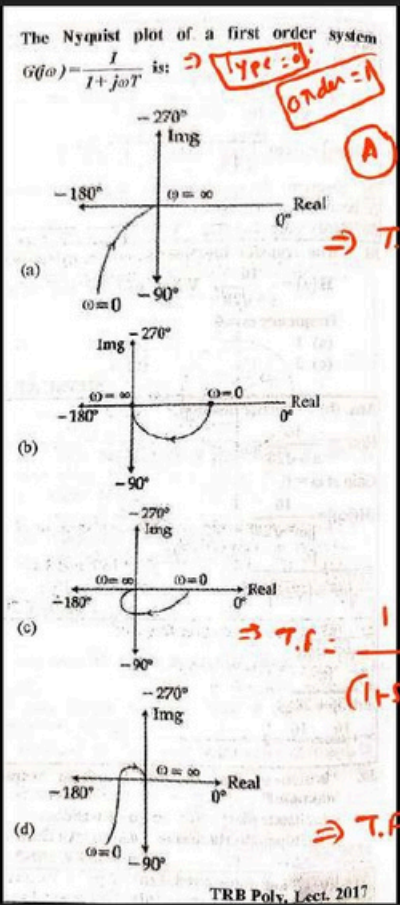
(a) There is no closed loop poles in  $R_{H+}$  of  $Z^+$  s-plane.

(b) There is 2 closed loop poles in  $R_{H+}$  of  $Z^+$  s-plane.

(D)

$Z_1 = 0$   
 $Z_1 \neq 0$





type: 0  
order: 1

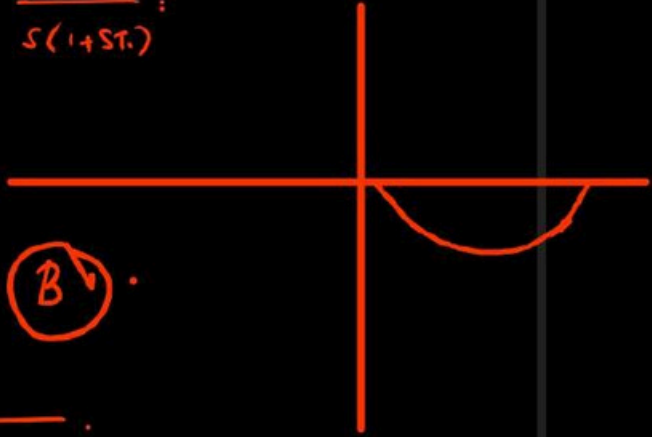
(A)

$$G(s)G(s) = \frac{1}{s+1+sT}$$

10%  $\Rightarrow$

90%  $\Rightarrow$

$\Rightarrow$  T.F. =  $\frac{1}{s(1+sT)}$



(B)

$\Rightarrow$  T.F. =  $\frac{1}{(1+sT_1)(1+sT_2)}$

$\Rightarrow$  T.F. =  $\frac{1}{s(1+sT_1)(1+sT_2)}$



# Compensators and Controllers.

## Compensators:

### 1. Lead Compensator:

→ High Pass Filter.

$$* \text{ T.F} = \frac{V_o(s)}{V_i(s)} = k \frac{(1 + s\tau)}{1 + \alpha s\tau}$$

$$\tau = R_1 C$$

$$\alpha = \frac{R_2}{R_1 + R_2}$$

Optimum  $\alpha = 0.1$   $\alpha < 1$

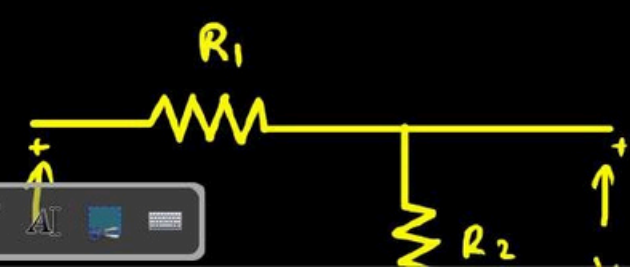
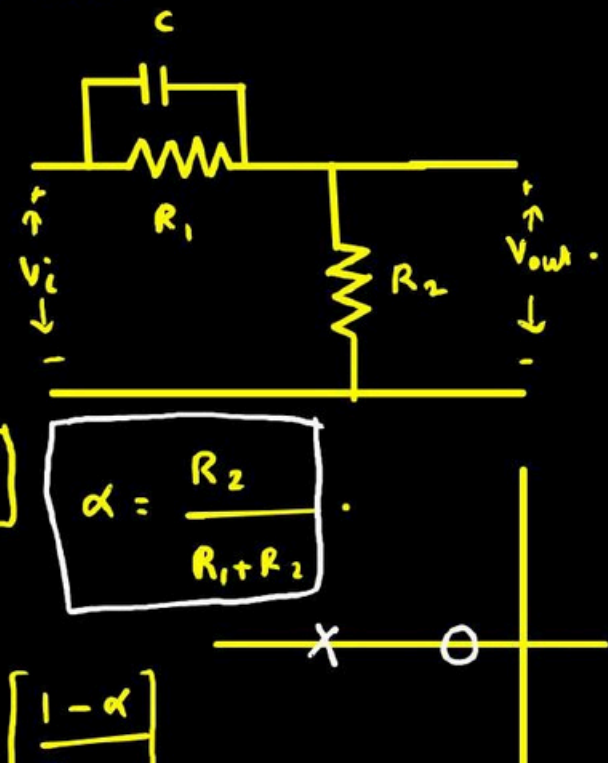
$$* \omega_{max} = \frac{1}{\tau} \sqrt{\frac{1}{\alpha}} = \sqrt{\omega_{c1} \omega_{c2}}$$

$$* \phi_{max} = \sin^{-1} \left[ \frac{1 - \alpha}{1 + \alpha} \right]$$

### 2. Lag Compensators.

→ Low Pass Filter.

$$\left( 1 + \frac{s}{\omega_{c1}} \right)$$



Full Video Link



0.7 x

# Compensators and Controllers.

## Compensators:

### 1. Lead Compensator:

→ High Pass Filter.

$$* \text{T.F} = \frac{V_o(s)}{V_i(s)} = k \frac{(1 + s\tau)}{1 + \alpha s\tau}$$

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Optimum  $\alpha = 0.1$   $\alpha < 1$

$$* \omega_{max} = \frac{1}{\tau} \sqrt{\frac{1}{\alpha}} = \sqrt{\omega_{c1} \omega_{c2}}$$

$$* \phi_{max} = \sin^{-1} \left[ \frac{1 - \alpha}{1 + \alpha} \right]$$

### 2. Lag Compensators.

→ Low Pass Filter.

$$\text{T.F} = \frac{V_o(s)}{V_i(s)} = k \frac{(1 + s\tau)}{1 + \beta s\tau}$$

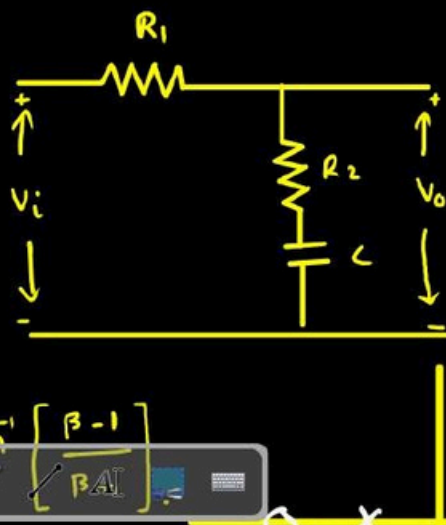
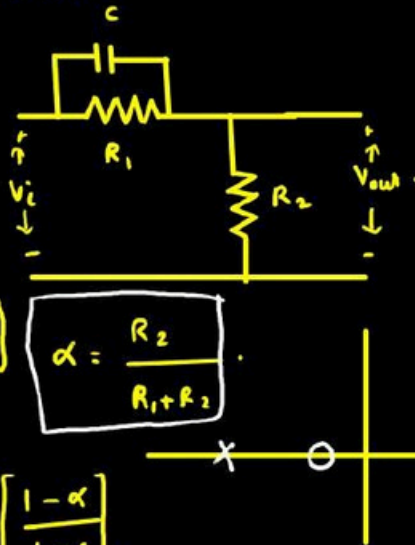
$$\beta > 1$$

$$\tau = R_2 C$$

$$\beta = \frac{R_1 + R_2}{R_2}$$

$$* \omega_{max} = \frac{1}{\tau} \sqrt{\frac{1}{\beta}}$$

$$* \phi_{max} = \sin^{-1} \left[ \frac{\beta - 1}{\beta + 1} \right]$$



## 2. Lag Compensators.

$$\hookrightarrow \left(1 + \frac{s}{\omega_{c1}}\right)$$

→ Low Pass Filter.

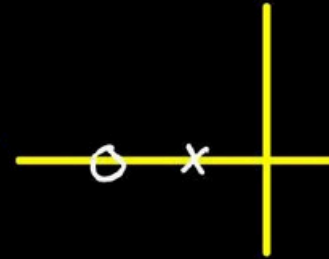
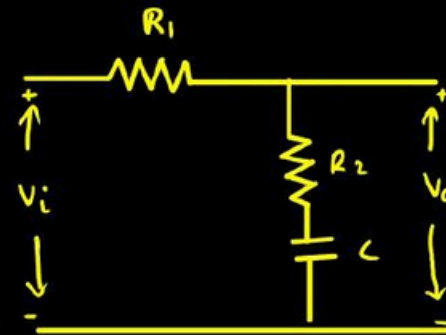
$$T.F = \frac{V_o(s)}{V_i(s)} = \frac{k(1+s\tau)}{1+\beta s\tau}$$

$$\boxed{\beta > 1} \cdot \boxed{\tau = R_2 C} \quad \boxed{\beta = \frac{R_1 + R_2}{R_2}}$$

$$* \omega_{max} = \frac{1}{\tau} \sqrt{\frac{1}{\beta}}$$

$$* \omega_{max} = \sqrt{\omega_{c1} \times \omega_{c2}}$$

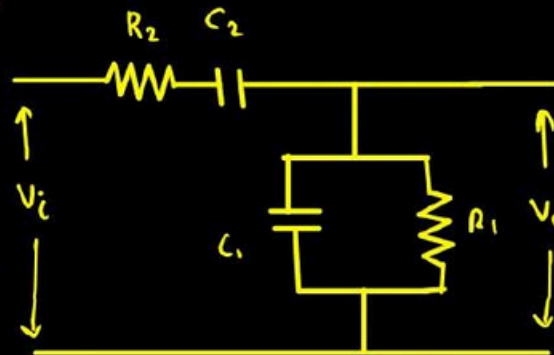
$$* \phi_{max} = \sin^{-1} \left[ \frac{\beta - 1}{\beta + 1} \right]$$



## 3. Lag-Lead Compensation:

$$T.F = \frac{(1+s\tau_1)(1+s\tau_2)}{(1+\alpha s\tau_1)(1+\beta s\tau_2)}$$

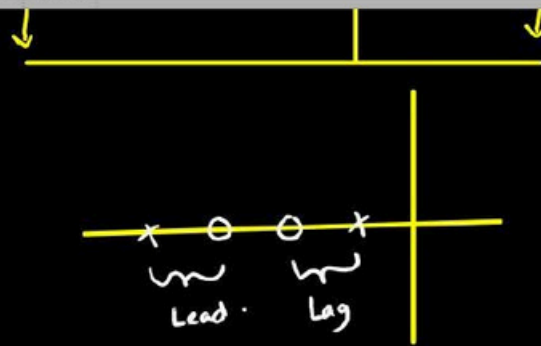
→ Band stop / Band Reject Filter.



Full Video Link



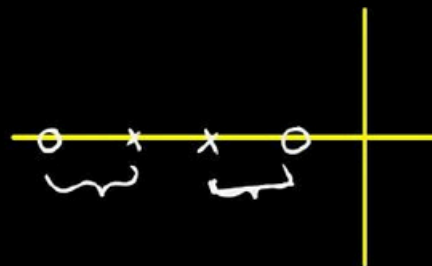
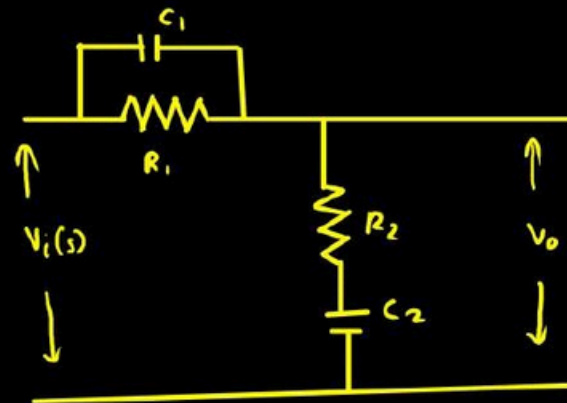
0.5 x



4. Lead-Lag Compensators.

$$T.F.: \frac{(1 + s\tau_1)(1 + s\tau_2)}{(1 + \alpha s\tau_1)(1 + \beta s\tau_2)}$$

→ Band Pass Filter.



Controllers. ( $t_r$  and  $t_{settle} \downarrow$ ;  $E_{ss} \downarrow$ ).

\* P Controller:

→ Change transient performance of a system.

Type (unaffected)  $E_{ss}$  (unaffected) Stability (unaffected).

$$G_c(s) = K_p.$$

\* I Controller:

$$G_c(s) = \frac{K_I}{s}.$$

→ Type  $\uparrow \Rightarrow E_{ss} \downarrow \Rightarrow$  stability  $\downarrow \Rightarrow$  Accuracy increased.

\* D Controller:

$$G_c(s) = K_D s.$$

→ Type  $\downarrow \Rightarrow E_{ss} \uparrow \Rightarrow$  stability  $\uparrow \Rightarrow$  Less Accuracy.

\* PI Controller [Lag].

Full Video Link



\* PI Controller [Lag].

$$G_c(s) = K_p + \frac{K_I}{s} = \frac{sk_p + k_I}{s}$$

→ Type ↑ ⇒  $E_{ss}$  ↓ ⇒ Stability ↓ But adding one zero  
Stability unaffected.

\* PD Controller [lead].

$$G_c(s) = K_p + K_D s$$

→ Type unaffected ⇒  $E_{ss}$  unaffected ⇒ Stability ↑.

\* PID Controller:

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s = \frac{s^2 K_D + K_p s + K_I}{s}$$

→ Type ↑ ⇒  $E_{ss}$  ↓ ⇒ Stability ↓ ⇒ But adding 2 zeroes.  
Stability ↑ ↘ unaffected → Stability ↑.

Full Video Link



A phase-lead network has its transfer function

$$G_C(s) = \frac{(1 + 0.04s)}{(1 + 0.01s)}$$

What is the frequency at which the maximum phase-lead occurs?

- (a) 25 rad/sec                      (b) 50 rad/sec  
(c) 75 rad/sec                      (d) 100 rad/sec

ESE 2017  
ESE 1996

(b)

$$\omega_{max} = \sqrt{25 \times 100} = 5 \times 10 = 50 \text{ rad/sec}$$

Q.1].  $\omega_{max} = \sqrt{\omega_{C1} \times \omega_{C2}} = \frac{1}{T} \sqrt{\frac{1}{\alpha}}$

$$G_C(s) = \frac{1 + 0.04s}{1 + 0.01s} \rightarrow \frac{s}{\omega_{C1}} = 0.04s \quad \omega_{C1} = \frac{1}{0.04} = \frac{25}{4}$$
$$\rightarrow \frac{s}{\omega_{C2}} = 0.01s \Rightarrow \omega_{C2} = \frac{1}{0.01} = 100$$



Full Video Link



**Transfer function of two compensators are:**

$$C_1 = \frac{100(S+3)}{(S+200)}; C_2 = \frac{S+200}{100(S+3)}$$

**Which of the following statement is correct?**

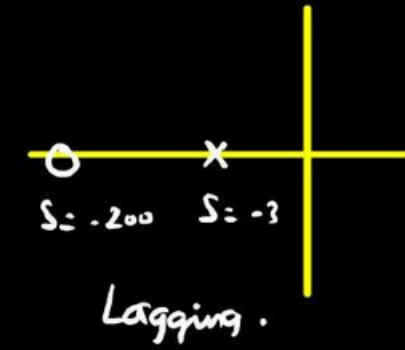
- (a)  $C_1$  - lead and  $C_2$  - lag
- (b)  $C_1$  - lag and  $C_2$  - lead
- (c) Both  $C_1$  and  $C_2$  are lag
- (d) Both  $C_1$  and  $C_2$  are lead

**ISRO Scientist/Engineer 2018**

$$C_2 = \frac{S+200}{100(S+3)}$$

Poles  $\Rightarrow S = -3$ .

Zeros  $\Rightarrow S = -200$ .

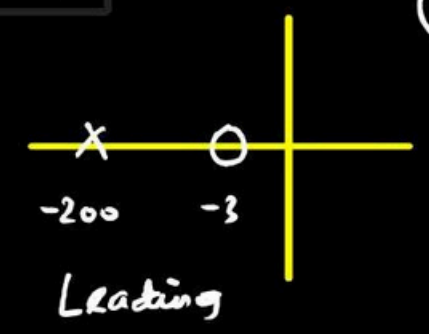


(A)

Q.2].  $C_1 = \frac{100(S+3)}{(S+200)}$

poles  $\Rightarrow S = -200$ .

Zeros  $\Rightarrow S = -3$ .



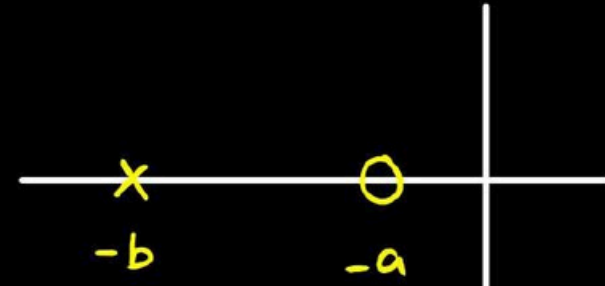
Transfer function of lead compensator used for

a closed loop controller is  $\frac{K \left(1 + \frac{s}{a}\right)}{\left(1 + \frac{s}{b}\right)}$  For such a

lead compensator:

- (a)  $a < Kb$                       (b)  $a > Kb$   
(c)  $a < b$                          (d)  $b < a$

KPSC AE 2014



$a > b$

(d)

Q.3]. Lead. poles  $\Rightarrow 1 + \frac{s}{b} = 0$ .

Zeros  $\Rightarrow 1 + \frac{s}{a} = 0$ .  $\frac{s}{b} = -1 \Rightarrow s = -b$ .

$\frac{s}{a} = -1 \Rightarrow s = -a$

Full Video Link



0.4 x

A system with transfer function

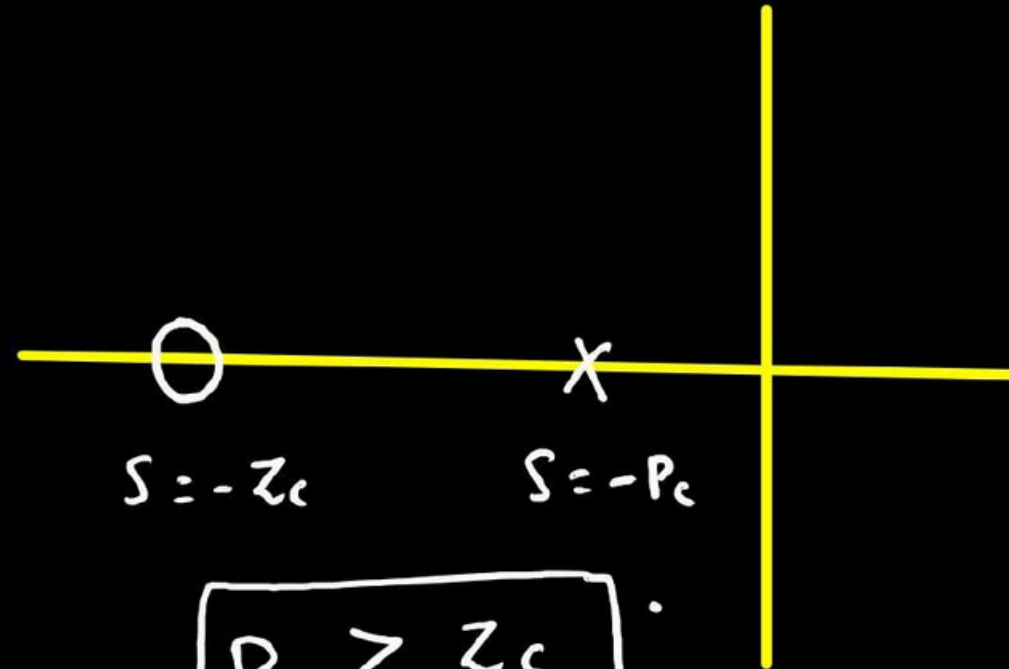
$$F(s) = K \frac{s + z_c}{s + p_c}$$

is used as Lag compensator to improve the steady error.

What conditions should be met to achieve this?

- (a)  $z_c > p_c$                       (b)  $z_c < p_c$   
(c)  $z_c = p_c$                       (d)  $p_c = 0$

UPRVUNL (UPJVNL) AE 2016



Q.4]. Lag compensator.

poles  $\Rightarrow s + p_c = 0 \Rightarrow s = -p_c$ .

zeros  $\Rightarrow s + z_c = 0 \Rightarrow s = -z_c$ .

(b)

$$p_c > z_c$$



The transfer function is  $(1 + 0.5s) / (1 + s)$ . It represents a

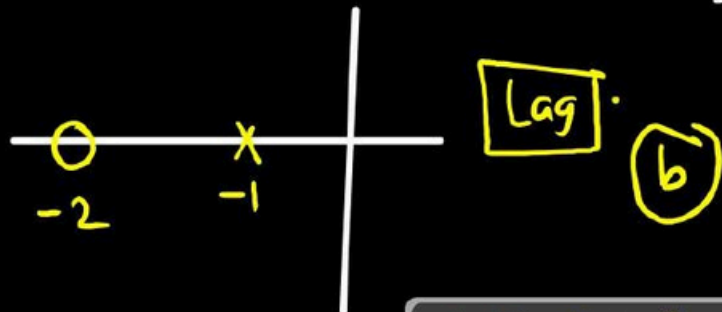
- (a) Lead network
- (b) Lag network
- (c) Lag-Lead network
- (d) Proportional controller

PTCUL AE 2012

Q.5]. T.F =  $\frac{1 + 0.5s}{1 + s}$ .

II Poles  $\Rightarrow 1 + s = 0 \Rightarrow s = -1$ .

Zeros  $\Rightarrow 1 + 0.5s = 0 \Rightarrow 0.5s = -1$   
 $s = \frac{-1}{0.5} = -\frac{1 \times 2}{1} = -2$ .



I. T.F =  $\frac{k(1 + s\tau)}{1 + \alpha s\tau}$ . (b)

$s\tau = 0.5s \Rightarrow \tau = 0.5$ .

$\alpha s\tau = s \Rightarrow \alpha = \frac{1}{\tau} = \frac{1}{0.5}$ .

$\alpha = 2$   
 $\frac{1}{\alpha} = \frac{1}{2} = 0.5$



The transfer function of an integral compensator is given by:

(a)  $\frac{1}{s^2}$  (b)  $\frac{1}{s}$

(c)  $\frac{K}{s}$  (d)  $KS$

UPPCL AE 18-05-2016

©

Q.6].  $G_c(s) = \frac{k_I}{s}$



Which of the following Transfer function is phase lead compensator?

(a)  $\frac{12S+6}{12S+4}$

(b)  $\frac{S+8}{S^2+5S+6}$

(c)  $\frac{2S+2}{2S+4}$

(d)  $\frac{2S+10}{6S+4}$

UPPCL AE 18-05-2016

©

T.F =  $\frac{2S+2}{2S+4}$

poles  $\Rightarrow 2S+4=0 \Rightarrow 2S=-4$

Zeros  $\Rightarrow 2S+2=0$

$\Rightarrow 2S=-2$

$S = -\frac{4}{2} = -2$

$S = -1$

Lead.

Q.7.

A)  $\frac{12S+6}{12S+4}$

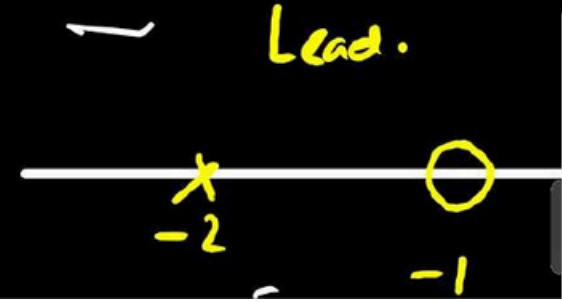
Lag

poles  $\Rightarrow 12S = -4$

$S = \frac{-4}{12} = -\frac{1}{3} = -0.33$

Zeros  $\Rightarrow 12S+6=0$

$12S = -6$



Which of the following Transfer function is phase lead compensator?

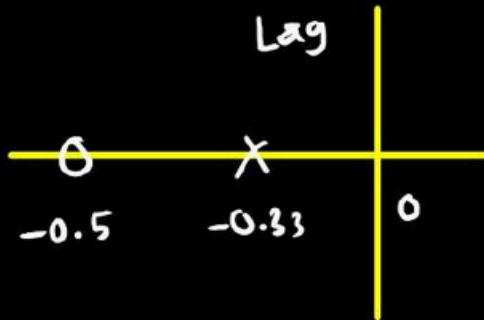
- (a)  $\frac{12S+6}{12S+4}$       (b)  $\frac{S+8}{S^2+5S+6}$   
 (c)  $\frac{2S+2}{2S+4}$       (d)  $\frac{6S+4}{6S+4}$

UPPCL AE 18-05-2016

Q.7.

A).  $\frac{12S+6}{12S+4}$

Lag



Poles  $\Rightarrow 12S = -4$

$S = \frac{-4}{12} = -\frac{1}{3} = -0.33$

Zeros  $\Rightarrow 12S + 6 = 0$

$12S = -6$

$S = \frac{-6}{12} = -\frac{1}{2} = -0.5$

C.

T.F =  $\frac{2S+2}{2S+4}$

$\frac{6}{3} \mid \frac{5}{2}$

Poles  $\Rightarrow 2S + 4 = 0 \Rightarrow 2S = -4$

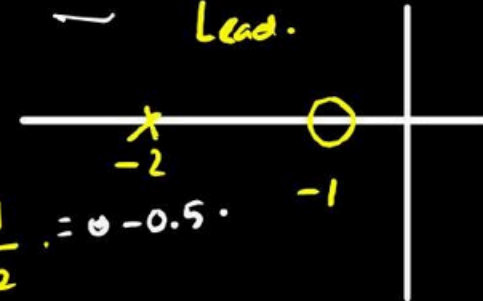
Zeros  $\Rightarrow 2S + 2 = 0$

$\Rightarrow 2S = -2$

$S = \frac{-4}{2} = -2$

$S = -1$

Lead.



Full Video Link



0.4 x

The maximum phase shift that can be provided by a lead compensator with transfer function

$$G(s) = \frac{1+6s}{1+2s} \text{ is}$$

- (a) 15°
- (b) 30°
- (c) 45°
- (d) 60°

UKPSC AE 2013 Paper-I  
 Vizag Steel MT 2012  
 UKPSC AE 2007, Paper-I

(b)

$$\begin{aligned} \phi_{max} &= \sin^{-1} \left[ \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} \right] \\ &= \sin^{-1} \left[ \frac{\frac{3-1}{3}}{\frac{3+1}{3}} \right] = \sin^{-1} \left[ \frac{\frac{2}{3}}{\frac{4}{3}} \right] \\ &= \sin^{-1} \left[ \frac{1}{2} \right] = 30^\circ \end{aligned}$$

Q.8].  $\phi_{max} = ?$   $\phi_{max} = \sin^{-1} \left[ \frac{1-\alpha}{1+\alpha} \right]$

$$G_c(s) = \frac{k [1+s\tau]}{1+\alpha s\tau}$$

$s\tau = 6s$      $\alpha s\tau = 2s$   
 $\tau = 6$      $\alpha = \frac{2}{6} = \frac{1}{3}$



Full Video Link



Compensator transfer function is given by,  $\frac{3s+9}{s+6}$ . The maximum phase lead in degree is

(a)  $26^\circ$  (b)  $18.45^\circ$   
 (c)  $15.9^\circ$  (d)  $19.47^\circ$

DSSSB AE 2019

Q.9.  $G(s) = \frac{K(1+s\tau)}{1+\alpha s\tau}$

T.F =  $\frac{9 \left[ 1 + \frac{3s}{43} \right]}{6 \left[ 1 + \frac{s}{2} \right]}$

$\tau = \frac{5}{8}$

$\alpha = \frac{8}{6}$

$\phi_{max} = \sin^{-1} \left[ \frac{1-\alpha}{1+\alpha} \right] = \sin^{-1} \left[ \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} \right]$

(d)

$\alpha = \frac{1}{2 \times \frac{1}{8}} = \frac{1}{2}$

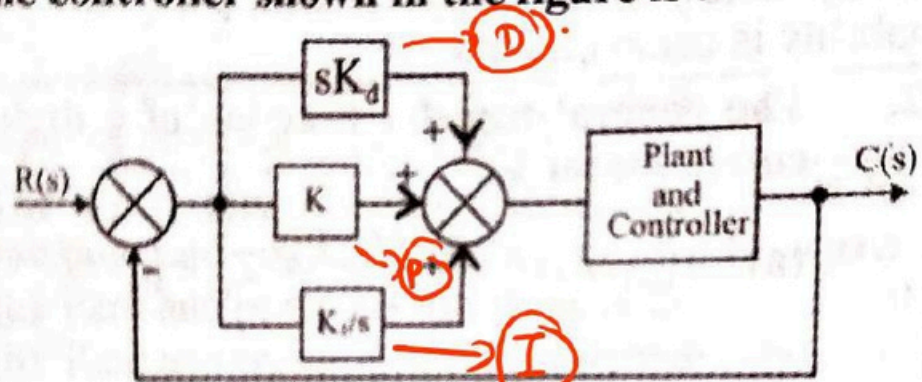
$= \sin^{-1} \left[ \frac{\frac{1}{2}}{\frac{3}{2}} \right]$

$= \sin^{-1} \left[ \frac{1}{3} \right]$

$\alpha = \frac{1}{6\tau} = 19.47^\circ$



The controller shown in the figure is :



- (a) PID type controller
- (b) P type controller
- (c) PI type controller
- (d) PD type controller

VPPCLAE 2019 OMC Deputy manager 2019

Full Video Link

